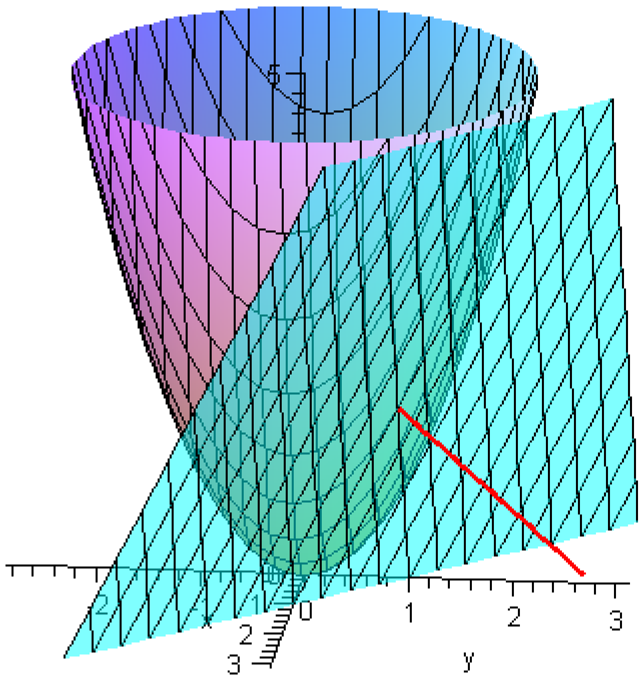
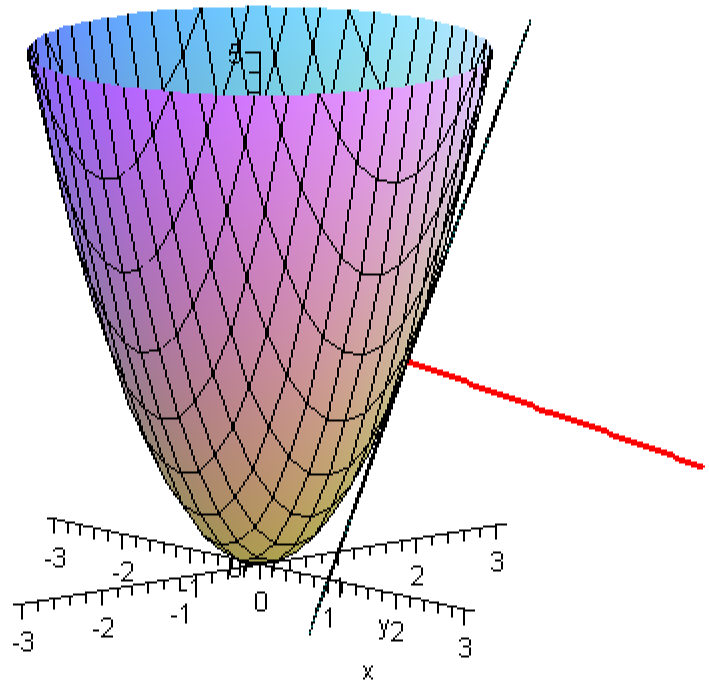
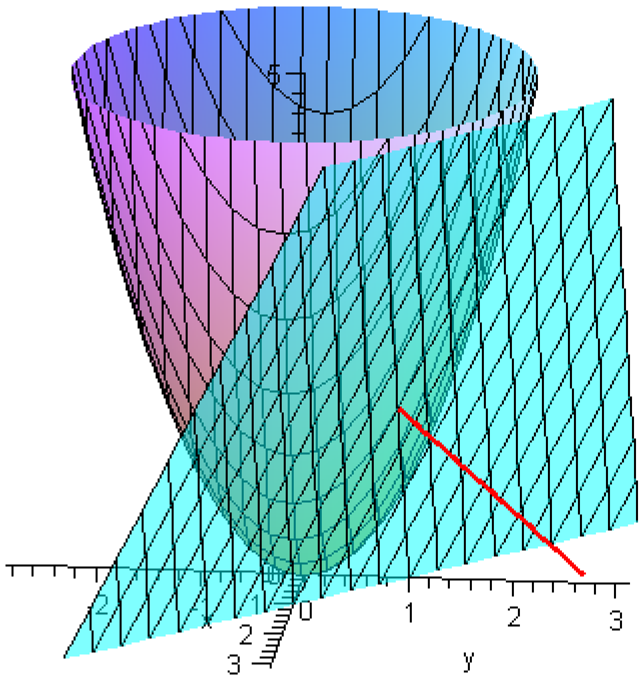
DIFFERENTIABILITY

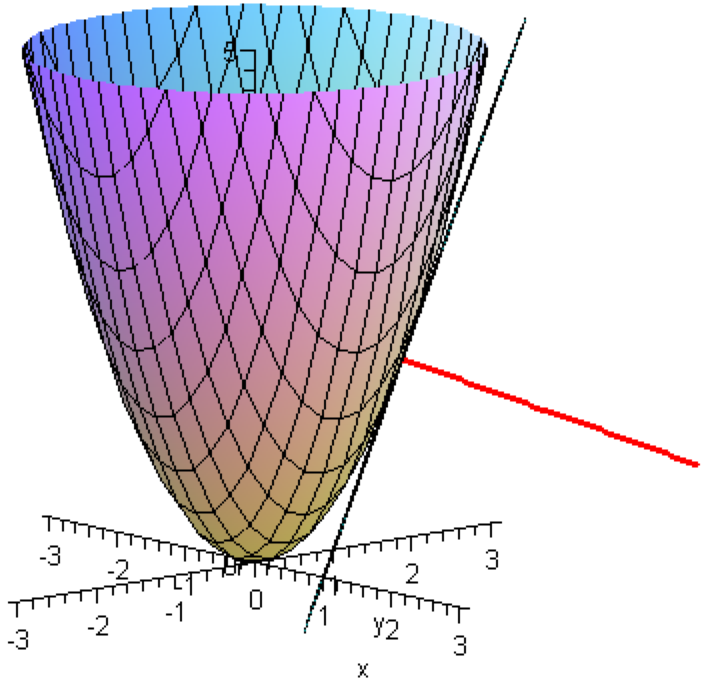
A function *z=f(x,y)* is differentiable at a point if a non-vertical tangent plane exists at that point.



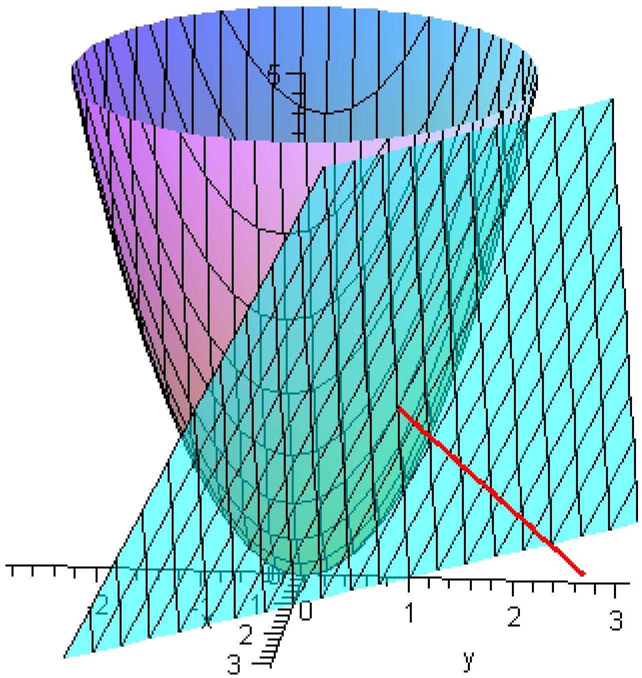


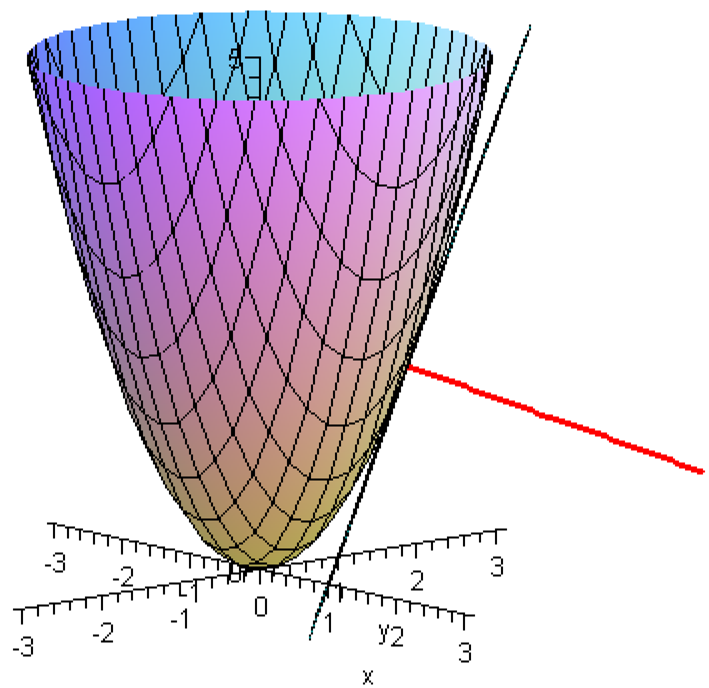
This means two important things.



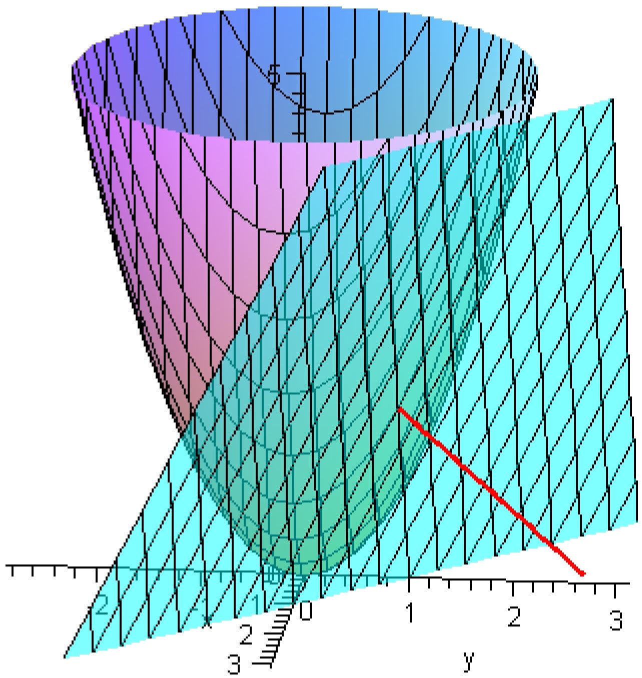


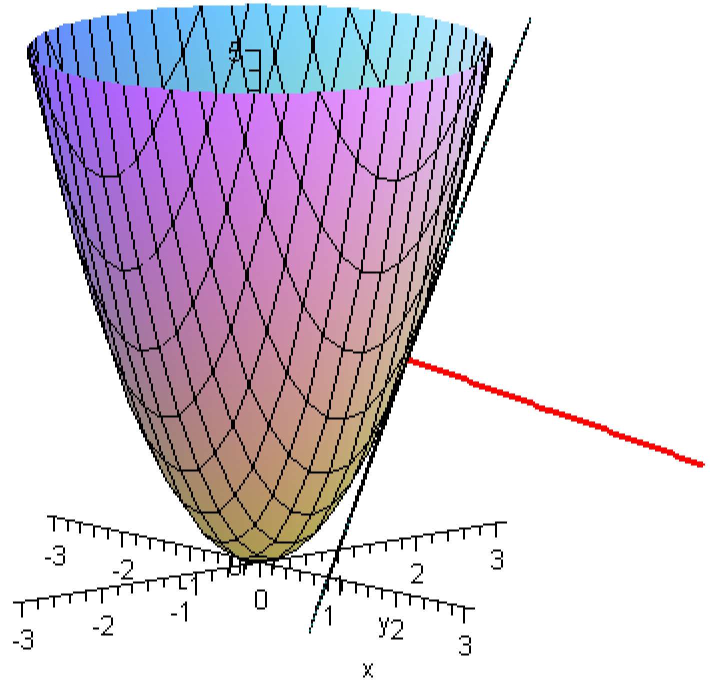
(1) We can define slopes of tangent lines in all directions.



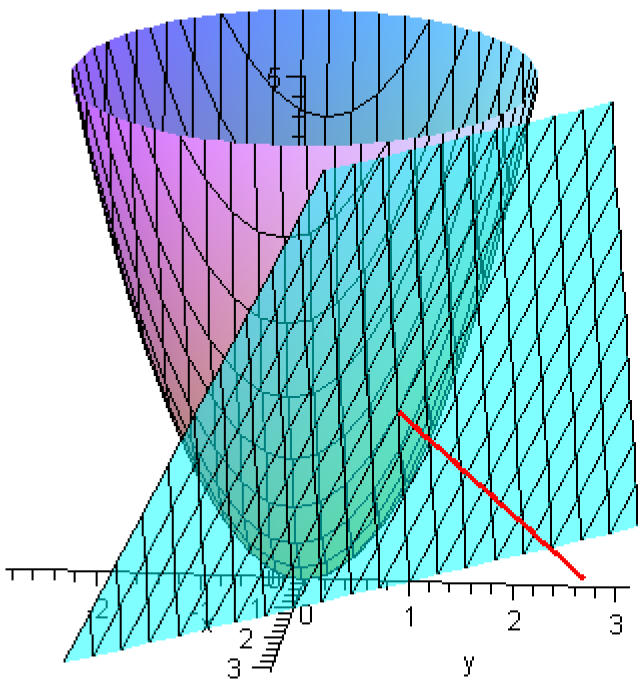


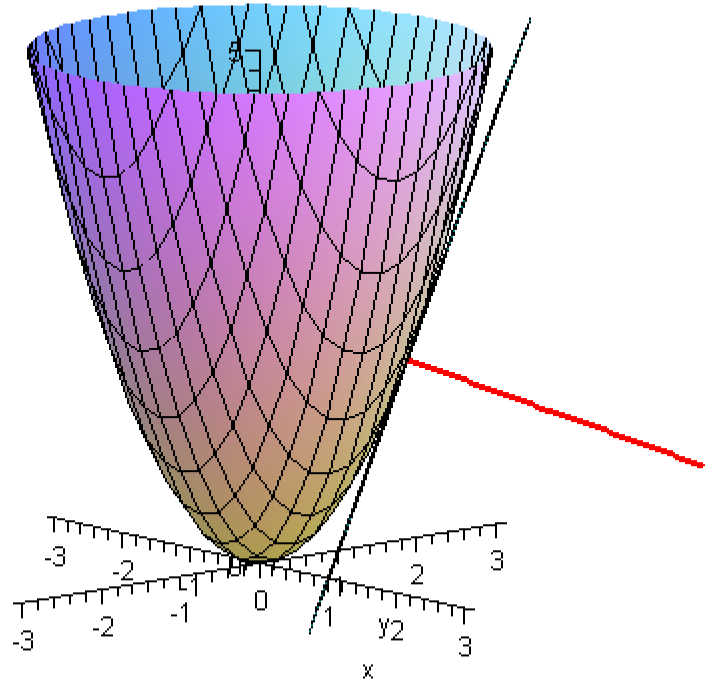
(2) The surface is locally linear at that point.





Local linearity means that as we zoom in on the point *(x,y)*, the surface resembles the tangent plane more and more.





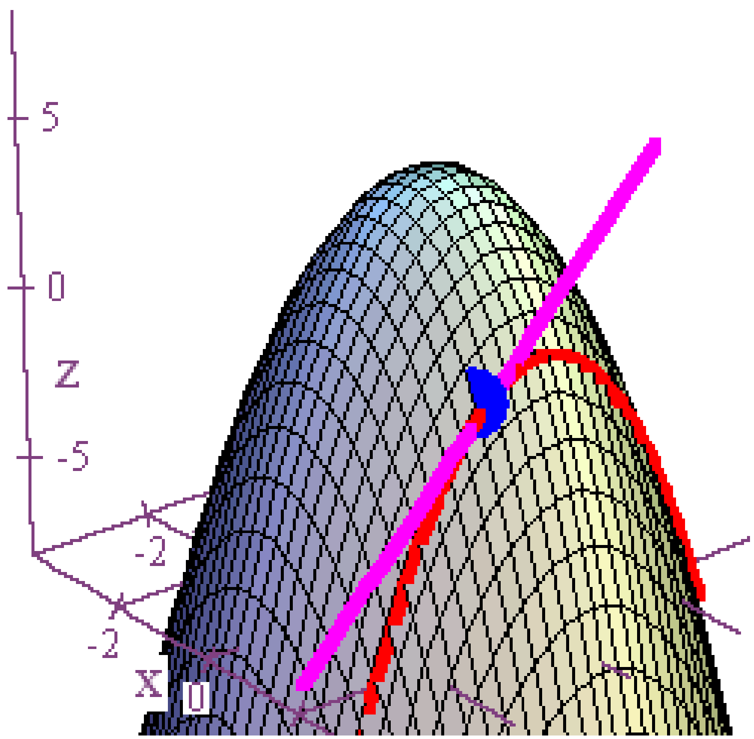
This latter condition is also often taken as a definition of differentiability

<u>Definition:</u> If *z=f(x,y)*, then *f* is differentiable at *(a,b)* if the change in *z* as we move away from *(*a,b) in any direction can be expressed in the form:

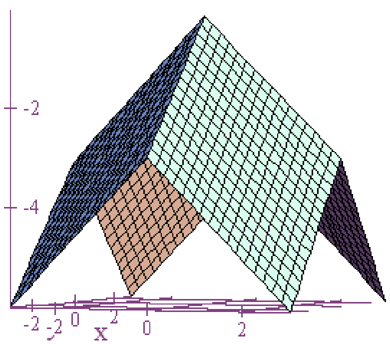
$$\Delta z = m_x \Delta x + m_y \Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$$

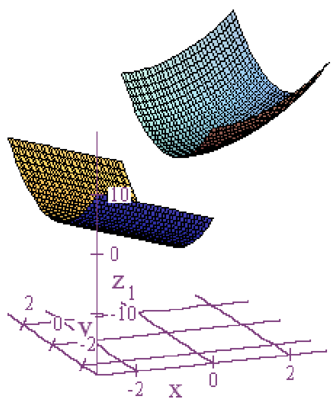
$$where\, \varepsilon_1, \varepsilon_2 \to 0\, as\, \Delta x, \Delta y \to 0.$$

Question: What sorts of things would prevent us from defining a tangent plane at a point on our surface?



Answer: The usual stuff. Sharp corner points and any sort of break in continuity.





Notice, also, that at the Grand Canyon, there are points where tangent lines exist in some directions but not others.