CHAIN RULE EXAMPLES

$$z = \ln(x^2 + y^2)$$

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$$y = \sqrt{t} = t^{1/2}$$

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$$\frac{dz}{dt} = \frac{2x}{x^2 + y^2} \cdot \frac{-1}{t^2} + \frac{2y}{x^2 + y^2} \cdot \frac{1}{2t^{1/2}}$$

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$$= \frac{\frac{2}{t}}{\left(\frac{1}{t}\right)^2 + \left(\sqrt{t}\right)^2} \cdot \frac{-1}{t^2} + \frac{2\sqrt{t}}{(1/t)^2 + \left(\sqrt{t}\right)^2} \cdot \frac{1}{2\sqrt{t}}$$

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$$\frac{dz}{dt} = \frac{1}{t^{-2} + t} \cdot \left(-2t^{-3} + 1\right)$$

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$$z = \ln(t^{-2} + t)$$

$$\frac{dz}{dt} = \frac{1}{t^{-2} + t} \cdot \left(-2t^{-3} + 1\right) = \frac{1}{\frac{1}{t^2} + t} \cdot \left(\frac{-2}{t^3} + 1\right)$$

$$= \frac{t^2}{1 + t^3} \cdot \left(\frac{-2 + t^3}{t^3}\right) = \frac{t^3 - 2}{t + t^4}$$

$$z = \tan^{-1}\left(\frac{x}{y}\right)$$

$$x = u^2 + v^2$$

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$$\frac{\partial z}{\partial u} = \frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\frac{1}{y}\cdot 2u + \frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\frac{-x}{y^2}\cdot 2u$$

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$$= \frac{y^2}{y^2+x^2}\cdot\frac{1}{y}\cdot 2u + \frac{y^2}{y^2+x^2}\cdot\frac{-x}{y^2}\cdot 2u = \frac{y-x}{y^2+x^2}\cdot 2u$$

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$$= \frac{(u^2-v^2)-(u^2+v^2)}{(u^2-v^2)^2+(u^2+v^2)^2}\cdot 2u = \frac{-2uv^2}{u^4+v^4}$$