POSITION, VELOCITY, AND ACCELERATION - ANSWERS

Find the position and velocity functions having the given acceleration and initial position and velocity at time $t = 0$ *seconds*. Then find the speed at time $t = 5$ *seconds*. Assume that the units associated with distance are *feet*.

1. $\vec{a}(t) = -32\hat{k},\ \vec{v}(0) = \vec{0},\ \vec{r}(0) = 100\hat{k}$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \left(\int -32\,dt\right)\hat{k} = (-32t)\hat{k} + \vec{C}$$
$$\vec{v}(0) = \vec{0} + \vec{C} = \vec{0} \Rightarrow \vec{C} = \vec{0}$$
$$\vec{v}(t) = (-32t)\hat{k}$$

$$\vec{r}(t) = \int \vec{v}(t)dt = \left(\int -32t\,dt\right)\vec{k} = (-16t^2)\hat{k} + \vec{C}$$
$$\vec{r}(0) = \vec{0} + \vec{C} = 100\hat{k} \Rightarrow \vec{C} = 100\hat{k}$$
$$\vec{r}(t) = (-16t^2)\hat{k} + 100\hat{k} = (-16t^2 + 100)\hat{k}$$

$$\text{speed } = \left\|\vec{v}(5)\right\| = \left\|-160\hat{k}\right\| = 160\frac{feet}{\text{sec}ond}$$

2. $\vec{a}(t) = -32\hat{k},\ \vec{v}(0) = -3\hat{k},\ \vec{r}(0) = 100\hat{k}$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \left(\int -32\,dt\right)\hat{k} = (-32t)\hat{k} + \vec{C}$$
$$\vec{v}(0) = \vec{0} + \vec{C} = -3\hat{k} \Rightarrow \vec{C} = -3\hat{k}$$
$$\vec{v}(t) = (-32t)\hat{k} - 3\hat{k} = (-32t - 3)\hat{k}$$

$$\vec{r}(t) = \int \vec{v}(t)dt = \left(\int -32t\,dt\right)\vec{k} = (-16t^2 - 3t)\hat{k} + \vec{C}$$
$$\vec{r}(0) = \vec{0} + \vec{C} = 100\hat{k} \Rightarrow \vec{C} = 100\hat{k}$$
$$\vec{r}(t) = (-16t^2 - 3t)\hat{k} + 100\hat{k} = (-16t^2 - 3t + 100)\hat{k}$$

$$\text{speed } = \left\|\vec{v}(5)\right\| = \left\|(-160 - 3)\hat{k}\right\| = 163\frac{feet}{\text{sec}ond}$$

3. $\vec{a}(t) = -32\hat{k},\ \vec{v}(0) = 10\hat{i} + 10\hat{j} + 10\hat{k},\ \vec{r}(0) = \vec{0}$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \left(\int -32\,dt\right)\hat{k} = (-32t)\hat{k} + \vec{C}$$
$$\vec{v}(0) = \vec{0} + \vec{C} = 10\hat{i} + 10\hat{j} + 10\hat{k} \Rightarrow \vec{C} = 10\hat{i} + 10\hat{j} + 10\hat{k}$$
$$\vec{v}(t) = (-32t)\hat{k} + 10\hat{i} + 10\hat{j} + 10\hat{k} = 10\hat{i} + 10\hat{j} + (-32t + 10)\hat{k}$$

$$\vec{r}(t) = \int \vec{v}(t)dt = \left(\int 10dt\right)\hat{i} + \left(\int 10dt\right)\hat{j} + \left(\int (-32t + 10)dt\right)\hat{k}$$
$$= (10t)\hat{i} + (10t)\hat{j} + (-16t^2 + 10t)\hat{k} + \vec{C}$$
$$\vec{r}(0) = \vec{0} + \vec{C} = \vec{0} \Rightarrow \vec{C} = \vec{0}$$
$$\vec{r}(t) = (10t)\hat{i} + (10t)\hat{j} + (-16t^2 + 10t)\hat{k}$$

$$\text{speed} = \left\|\vec{v}(5)\right\| = \left\|10\hat{i} + 10\hat{j} - 150\hat{k}\right\| = \sqrt{22{,}700} = 10\sqrt{227}\,\frac{feet}{\text{sec}ond} \approx 150.67\,\frac{feet}{\text{sec}ond}$$

4. $\vec{a}(t) = -\cos(t)\hat{i} - \sin(t)\hat{j},\ \vec{v}(0) = -\hat{i},\ \vec{r}(0) = \hat{j}$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \left(\int -\cos(t)\,dt\right)\hat{i} + \left(\int -\sin(t)dt\right)\hat{j} = -\sin(t)\hat{i} + \cos(t)\hat{j} + \vec{C}$$
$$\vec{v}(0) = \hat{j} + \vec{C} = -\hat{i} \Rightarrow \vec{C} = -\hat{i} - \hat{j}$$
$$\vec{v}(t) = \left(-\sin(t) - 1\right)\hat{i} + \left(\cos(t) - 1\right)\hat{j}$$

$$\vec{r}(t) = \int \vec{v}(t)dt = \left(\int \left(-\sin(t) - 1\right)dt\right)\hat{i} + \left(\int \left(\cos(t) - 1\right)dt\right)\hat{j} = \left(\cos(t) - t\right)\hat{i} + \left(\sin(t) - t\right)\hat{j} + \vec{C}$$
$$\vec{r}(0) = \hat{i} + \vec{C} = \hat{j} \Rightarrow \vec{C} = -\hat{i} + \hat{j}$$
$$\vec{r}(t) = \left(\cos(t) - t\right)\hat{i} + \left(\sin(t) - t\right)\hat{j} - \hat{i} + \hat{j} = \left(\cos(t) - t - 1\right)\hat{i} + \left(\sin(t) - t + 1\right)\hat{j}$$

$$\text{speed} = \left\|\vec{v}(5)\right\| = \left\|\left(-\sin(5) - 1\right)\hat{i} + \left(\cos(5) - 1\right)\hat{j}\right\| = \sqrt{\left(-\sin(5) - 1\right)^2 + \left(\cos(5) - 1\right)^2}$$
$$\approx 0.7175\,\frac{feet}{\text{sec}ond}$$

5. $\vec{a}(t) = -5\cos(t)\hat{i} - 5\sin(t)\hat{j},\ \vec{v}(0) = -5\hat{i} + 5\hat{j},\ \vec{r}(0) = 5\hat{i} + 5\hat{j}$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \left(\int -5\cos(t)\,dt\right)\hat{i} + \left(\int -5\sin(t)dt\right)\hat{j} = -5\sin(t)\hat{i} + 5\cos(t)\hat{j} + \vec{C}$$
$$\vec{v}(0) = 5\hat{j} + \vec{C} = -5\hat{i} + 5\hat{j} \Rightarrow \vec{C} = -5\hat{i}$$
$$\vec{v}(t) = \left(-5\sin(t) - 5\right)\hat{i} + 5\cos(t)\hat{j}$$

$$\vec{r}(t) = \int \vec{v}(t)dt = \left(\int \left(-\sin(t) - 5\right)dt\right)\hat{i} + \left(\int 5\cos(t)dt\right)\hat{j} = \left(5\cos(t) - 5t\right)\hat{i} + 5\sin(t)\hat{j} + \vec{C}$$
$$\vec{r}(0) = 5\hat{i} + \vec{C} = 5\hat{i} + 5\hat{j} \Rightarrow \vec{C} = 5\hat{j}$$
$$\vec{r}(t) = \left(5\cos(t) - 5t\right)\hat{i} + 5\sin(t)\hat{j} + 5\hat{j} = \left(5\cos(t) - 5t\right)\hat{i} + \left(5\sin(t) + 5\right)\hat{j}$$

$$\text{speed } = \left\|\vec{v}(5)\right\| = \left\|\left(-5\sin(5) - 5\right)\hat{i} + 5\cos(5)\hat{j}\right\| = \sqrt{\left(-5\sin(5) - 5\right)^2 + 25\cos^2 5}$$
$$\approx 1.4331 \frac{feet}{\text{sec}ond}$$