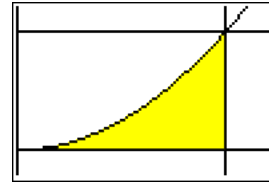


ORDER OF INTEGRATION - ANSWERS

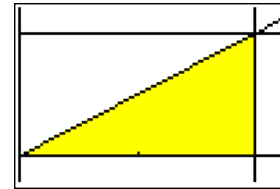
Evaluate the following integrals by reversing the order of integration.

$$1. \int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1} \, dx dy$$



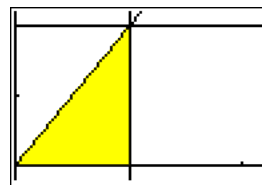
$$\begin{aligned} \int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1} \, dx dy &= \int_0^1 \int_0^{x^2} \sqrt{x^3+1} \, dy dx = \int_0^1 y \sqrt{x^3+1} \Big|_0^{x^2} dx \\ &= \int_0^1 x^2 \sqrt{x^3+1} \, dx = \frac{2(x^3+1)^{3/2}}{9} \Big|_0^1 = \frac{2 \cdot 2^{3/2}}{9} - \frac{2}{9} = \frac{4\sqrt{2}-2}{9} \end{aligned}$$

$$2. \int_0^1 \int_{2y}^2 \sqrt{x^2+1} \, dx dy$$



$$\begin{aligned} \int_0^1 \int_{2y}^2 \sqrt{x^2+1} \, dx dy &= \int_0^2 \int_0^{x/2} \sqrt{x^2+1} \, dy dx = \int_0^2 y \sqrt{x^2+1} \Big|_0^{x/2} dx \\ &= \int_0^2 \frac{x}{2} \sqrt{x^2+1} \, dx = \frac{(x^2+1)^{3/2}}{6} \Big|_0^2 = \frac{5^{3/2}}{6} - \frac{1}{6} = \frac{5\sqrt{5}-1}{6} \end{aligned}$$

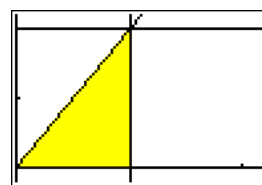
$$3. \int_0^2 \int_{\frac{y}{2}}^1 \sin(x^2) \, dx dy$$



$$\int_0^2 \int_{\frac{y}{2}}^1 \sin(x^2) \, dx dy = \int_0^1 \int_0^{2x} \sin(x^2) \, dy dx = \int_0^1 y \sin(x^2) \Big|_0^{2x} \, dx$$

$$= \int_0^1 2x \sin(x^2) \, dx = -\cos(x^2) \Big|_0^1 = -\cos(1) + \cos(0) = 1 - \cos(1) \approx 0.4597$$

$$4. \int_0^2 \int_{\frac{y}{2}}^1 e^{x^2} \, dx dy$$



$$\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2} \, dx dy = \int_0^1 \int_0^{2x} e^{x^2} \, dy dx = \int_0^1 y e^{x^2} \Big|_0^{2x} \, dx = \int_0^1 2x e^{x^2} \, dx$$

$$= e^{x^2} \Big|_0^1 = e - 1$$