INDEPENDENCE OF PATH - ANSWERS

For each vector field below, evaluate $\int_C \vec{F} \bullet d\vec{r}$ where C is any path from $(0,0)$ to $(1,1)$.

1. $\vec{F} = x\hat{i} + y\hat{j}$

$$\int_C \vec{F} \bullet d\vec{r} = \int_{(0,0)}^{(1,1)} \vec{F} \bullet d\vec{r} = \frac{x^2}{2} + \frac{y^2}{2}\Big|_{(0,0)}^{(1,1)} = \frac{1}{2} + \frac{1}{2} = 1$$

2. $\vec{F} = y\hat{i} + x\hat{j}$

$$\int_C \vec{F} \bullet d\vec{r} = \int_{(0,0)}^{(1,1)} \vec{F} \bullet d\vec{r} = xy\Big|_{(0,0)}^{(1,1)} = (1)(1) = 1$$

3. $\vec{F} = \cos(x)\hat{i} + \sin(y)\,\hat{j}$

$$\int_C \vec{F} \bullet d\vec{r} = \int_{(0,0)}^{(1,1)} \vec{F} \bullet d\vec{r} = \left[\sin x - \cos y\right]\Big|_{(0,0)}^{(1,1)} = \sin(1) - \cos(1) + 1$$

4. $\vec{F} = (e^x + y^2)\hat{i} + (\cos y + 2xy)\,\hat{j}$

$$\int_C \vec{F} \bullet d\vec{r} = \int_{(0,0)}^{(1,1)} \vec{F} \bullet d\vec{r} = e^x + xy^2 + \sin x\Big|_{(0,0)}^{(1,1)} = e + 1 + \sin(1) - 1 - 0 - 0 = e + \sin(1)$$

5. $\vec{F} = (3xy^2 + 5)\hat{i} + (3 + 3x^2 y)\,\hat{j}$

$$\int_C \vec{F} \bullet d\vec{r} = \int_{(0,0)}^{(1,1)} \vec{F} \bullet d\vec{r} = \frac{3x^2 y^2}{2} + 5x + 3y\Big|_{(0,0)}^{(1,1)} = \frac{3}{2} + 5 + 3 = \frac{19}{2}$$