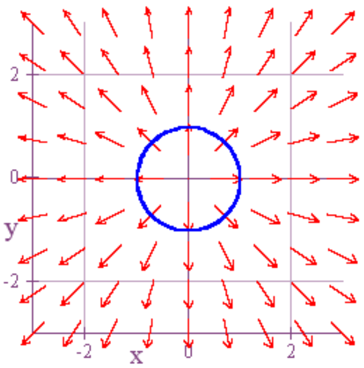
GREENS THEOREM AND STOKES THEOREM - ANSWERS

Use Green's Theorem (which in 2-dimensions is the same as Stoke's Theorem),

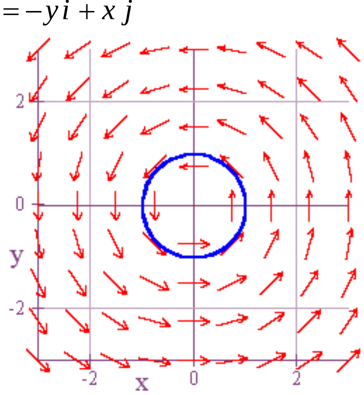
$Circulation = \int_C \vec{F} \bullet d\vec{r} = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_R \left(\nabla \times \vec{F} \right) \bullet \hat{k} \, dA$, to measure the circulation around the boundary of the unit circle (oriented counterclockwise) caused by each of the following vector fields.

1. $\vec{F} = x\hat{i} + y\hat{j}$



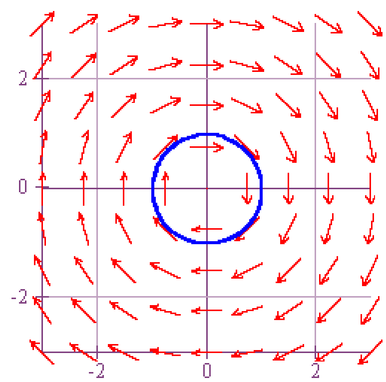
$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_R (0-0)\, dA = 0$$

2. $\vec{F} = -y\hat{i} + x\hat{j}$



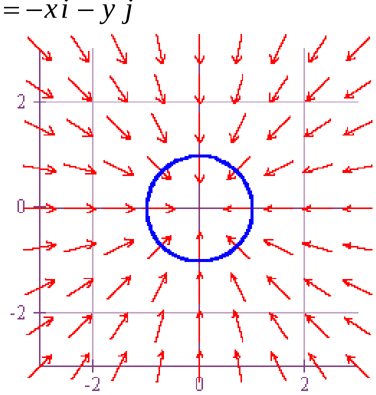
$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_R (1-(-1))\, dA = 2\pi$$

3. $\vec{F} = y\hat{i} - x\hat{j}$



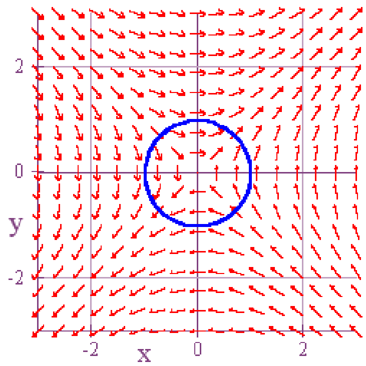
$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_R (-1-1)\, dA = -2\pi$$

4. $\vec{F} = -x\hat{i} - y\hat{j}$



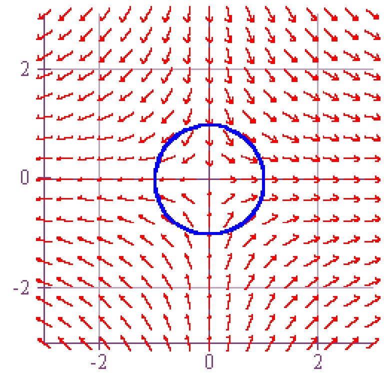
$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_R (0-0)\, dA = 0$$

5. $\vec{F} = y\hat{i} + x\hat{j}$



$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_R (1-1)\, dA = 0$$

6. $\vec{F} = 4x\hat{i} - 3y\hat{j}$



$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_R (0-0)\, dA = 0$$