

GRADIENT TO A LEVEL CURVE - ANSWERS

For each of the following functions, find ∇_z , the gradient.

1. $z = f(x, y) = x^3 y^2$

$$\nabla_z = \frac{\partial z}{\partial x} \hat{i} + \frac{\partial z}{\partial y} \hat{j} = 3x^2 y^2 \hat{i} + 2x^3 y \hat{j} = \langle 3x^2 y^2, 2x^3 y \rangle$$

2. $z = f(x, y) = \sin(x^3 y^2)$

$$\begin{aligned} \nabla_z &= \frac{\partial z}{\partial x} \hat{i} + \frac{\partial z}{\partial y} \hat{j} = \cos(x^3 y^2) \cdot 3x^2 y^2 \hat{i} + \cos(x^3 y^2) \cdot 2x^3 y \hat{j} \\ &= \langle 3\cos(x^3 y^2)x^2 y^2, 2\cos(x^3 y^2)x^3 y \rangle \end{aligned}$$

3. $z = f(x, y) = \sqrt{x^3 y^2}$

$$\begin{aligned} \nabla_z &= \frac{\partial z}{\partial x} \hat{i} + \frac{\partial z}{\partial y} \hat{j} = \frac{1}{2\sqrt{x^3 y^2}} \cdot 3x^2 y^2 \hat{i} + \frac{1}{2\sqrt{x^3 y^2}} \cdot 2x^3 y \hat{j} \\ &= \left\langle 3 \frac{1}{2\sqrt{x^3 y^2}} x^2 y^2, 2 \frac{1}{2\sqrt{x^3 y^2}} x^3 y \right\rangle = \left\langle \frac{3x^2 y^2}{2\sqrt{x^3 y^2}}, \frac{x^3 y}{\sqrt{x^3 y^2}} \right\rangle \end{aligned}$$

4. $z = f(x, y) = \sec(x^3 y^2)$

$$\begin{aligned} \nabla_z &= \frac{\partial z}{\partial x} \hat{i} + \frac{\partial z}{\partial y} \hat{j} = \sec(x^3 y^2) \tan(x^3 y^2) \cdot 3x^2 y^2 \hat{i} + \sec(x^3 y^2) \tan(x^3 y^2) \cdot 2x^3 y \hat{j} \\ &= \langle 3\sec(x^3 y^2) \tan(x^3 y^2)x^2 y^2, 2\sec(x^3 y^2) \tan(x^3 y^2)x^3 y \rangle \end{aligned}$$

5. $z = f(x, y) = \tan(x^3 y^2)$

$$\begin{aligned} \nabla_z &= \frac{\partial z}{\partial x} \hat{i} + \frac{\partial z}{\partial y} \hat{j} = \sec^2(x^3 y^2) \cdot 3x^2 y^2 \hat{i} + \sec^2(x^3 y^2) \cdot 2x^3 y \hat{j} \\ &= \langle 3\sec^2(x^3 y^2)x^2 y^2, 2\sec^2(x^3 y^2)x^3 y \rangle \end{aligned}$$