

GRADIENTS AND TANGENT PLANES - ANSWERS

For each of the following functions, use a gradient vector to find the equation of the tangent plane at the point $(1, 1, f(1, 1))$.

1. $z = f(x, y) = x^3 y^2$

$$P = (1, 1, 1)$$

$$w = x^3 y^2 - z$$

$$\nabla w = \frac{\partial w}{\partial x} \hat{i} + \frac{\partial w}{\partial y} \hat{j} + \frac{\partial w}{\partial z} \hat{k} = 3x^2 y^2 \hat{i} + 2x^3 y \hat{j} - \hat{k} = \langle 3x^2 y^2, 2x^3 y, -1 \rangle$$

$$\nabla w(1, 1, 1) = \langle 3, 2, -1 \rangle$$

$$3(x-1) + 2(y-1) - 1(z-1) = 0 \Rightarrow 3x + 2y - z - 4 = 0$$

$$\Rightarrow z = 3x + 2y - 4$$

2. $z = f(x, y) = \sin(x^3 y^2)$

$$P = (1, 1, \sin(1)) \approx (1, 1, .84)$$

$$w = f(x, y) - z = \sin(x^3 y^2) - z$$

$$\begin{aligned} \nabla w &= \frac{\partial w}{\partial x} \hat{i} + \frac{\partial w}{\partial y} \hat{j} + \frac{\partial w}{\partial z} \hat{k} = \cos(x^3 y^2) \cdot 3x^2 y^2 \hat{i} + \cos(x^3 y^2) \cdot 2x^3 y \hat{j} - \hat{k} \\ &= \langle 3\cos(x^3 y^2)x^2 y^2, 2\cos(x^3 y^2)x^3 y, -1 \rangle \end{aligned}$$

$$\nabla w(1, 1, .84) = \langle 3\cos(1), 2\cos(1), -1 \rangle \approx \langle 1.62, 1.08, -1 \rangle$$

$$1.62(x-1) + 1.08(y-1) - 1(z-.84) = 0 \Rightarrow 1.62x + 1.08y - z - 1.86 = 0$$

$$\Rightarrow z = 1.62x + 1.08y - 1.86$$

$$3. \quad z = f(x, y) = \sqrt{x^3 y^2}$$

$$P = (1, 1, 1)$$

$$w = f(x, y) - z = \sqrt{x^3 y^2} - z$$

$$\begin{aligned} \nabla w &= \frac{\partial w}{\partial x} \hat{i} + \frac{\partial w}{\partial y} \hat{j} + \frac{\partial w}{\partial z} \hat{k} = \frac{1}{2\sqrt{x^3 y^2}} \cdot 3x^2 y^2 \hat{i} + \frac{1}{2\sqrt{x^3 y^2}} \cdot 2x^3 y \hat{j} - \hat{k} \\ &= \left\langle 3 \frac{1}{2\sqrt{x^3 y^2}} x^2 y^2, 2 \frac{1}{2\sqrt{x^3 y^2}} x^3 y, -1 \right\rangle = \left\langle \frac{3x^2 y^2}{2\sqrt{x^3 y^2}}, \frac{x^3 y}{\sqrt{x^3 y^2}}, -1 \right\rangle \end{aligned}$$

$$\nabla w(1, 1, 1) = \left\langle \frac{3}{2}, 1, -1 \right\rangle$$

$$\begin{aligned} \frac{3}{2}(x-1) + 1(y-1) - 1(z-1) &= 0 \Rightarrow \frac{3}{2}x + y - z - \frac{3}{2} = 0 \\ \Rightarrow z &= \frac{3}{2}x + y - \frac{3}{2} \end{aligned}$$

$$4. \quad z = f(x, y) = \sec(x^3 y^2)$$

$$P = (1, 1, \sec(1)) \approx (1, 1, 1.85)$$

$$w = f(x, y) - z = \sec(x^3 y^2) - z$$

$$\begin{aligned} \nabla w &= \frac{\partial w}{\partial x} \hat{i} + \frac{\partial w}{\partial y} \hat{j} + \frac{\partial w}{\partial z} \hat{k} = \sec(x^3 y^2) \tan(x^3 y^2) \cdot 3x^2 y^2 \hat{i} + \sec(x^3 y^2) \tan(x^3 y^2) \cdot 2x^3 y \hat{j} - \hat{k} \\ &= \left\langle 3 \sec(x^3 y^2) \tan(x^3 y^2) x^2 y^2, 2 \sec(x^3 y^2) \tan(x^3 y^2) x^3 y, -1 \right\rangle \end{aligned}$$

$$\nabla w(1, 1, 1.85) = \left\langle 3 \sec(1) \tan(1), 2 \sec(1) \tan(1), -1 \right\rangle \approx \left\langle 8.65, 5.76, -1 \right\rangle$$

$$\begin{aligned} 8.65(x-1) + 5.76(y-1) - 1(z-1.85) &= 0 \Rightarrow 8.65x + 5.76y - z - 12.56 = 0 \\ \Rightarrow z &= 8.65x + 5.76y - 12.56 \end{aligned}$$

$$5. \quad z = f(x, y) = \tan(x^3 y^2)$$

$$P = (1, 1, \tan(1)) \approx (1, 1, 1.56)$$

$$w = f(x, y) - z = \tan(x^3 y^2) - z$$

$$\begin{aligned} \nabla w &= \frac{\partial w}{\partial x} \hat{i} + \frac{\partial w}{\partial y} \hat{j} + \frac{\partial w}{\partial z} \hat{k} = \sec^2(x^3 y^2) \cdot 3x^2 y^2 \hat{i} + \sec^2(x^3 y^2) \cdot 2x^3 y \hat{j} - \hat{k} \\ &= \left\langle 3 \sec^2(x^3 y^2) x^2 y^2, 2 \sec^2(x^3 y^2) x^3 y, -1 \right\rangle \end{aligned}$$

$$\nabla w(1, 1, 1.56) = \left\langle 3 \sec^2(1), 2 \sec^2(1), -1 \right\rangle \approx \left\langle 10.28, 6.85, -1 \right\rangle$$

$$\begin{aligned} 10.28(x-1) + 6.85(y-1) - 1(z-1.56) &= 0 \Rightarrow 10.28x + 6.85y - z - 15.57 = 0 \\ \Rightarrow z &= 10.28x + 6.85y - 15.57 \end{aligned}$$