GRADIENT FIELDS - ANSWERS

Verify that each vector field below is a conservative or gradient vector field by showing that $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$, and then find a potential function $z = f(x, y)$.

1. $\vec{F} = x\hat{i} + y\hat{j}$

$$\frac{\partial P}{\partial y} = 0 = \frac{\partial Q}{\partial x}$$

$$f(x, y) = \frac{x^2}{2} + g(y) \Rightarrow \frac{\partial f}{\partial y} = g'(y) = y \Rightarrow g(y) = \frac{y^2}{2} \Rightarrow z = f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$$

2. $\vec{F} = y\hat{i} + x\hat{j}$

$$\frac{\partial P}{\partial y} = 1 = \frac{\partial Q}{\partial x}$$

$$f(x, y) = xy + g(y) \Rightarrow \frac{\partial f}{\partial y} = x + g'(y) = x \Rightarrow g(y) = 0 \Rightarrow z = f(x, y) = xy$$

3. $\vec{F} = \cos(x)\hat{i} + \sin(y)\hat{j}$

$$\frac{\partial P}{\partial y} = 0 = \frac{\partial Q}{\partial x}$$

$$f(x, y) = \sin(x) + g(y) \Rightarrow \frac{\partial f}{\partial y} = g'(y) = \sin(y) \Rightarrow g(y) = -\cos(y)$$

$$\Rightarrow z = f(x, y) = \sin(x) - \cos(y)$$

4. $\vec{F} = (e^x + y^2)\hat{i} + (\cos y + 2xy)\hat{j}$

$$\frac{\partial P}{\partial y} = 2y = \frac{\partial Q}{\partial x}$$

$$f(x,y) = e^x + xy^2 + g(y) \Rightarrow \frac{\partial f}{\partial y} = 2xy + g'(y) = \cos y + 2xy \Rightarrow g(y) = \sin y$$

$$\Rightarrow z = f(x,y) = e^x + xy^2 + \sin y$$

5. $\vec{F} = (3xy^2 + 5)\hat{i} + (3 + 3x^2 y)\hat{j}$

$$\frac{\partial P}{\partial y} = 6xy = \frac{\partial Q}{\partial x}$$

$$f(x,y) = \frac{3x^2 y^2}{2} + 5x + g(y) \Rightarrow \frac{\partial f}{\partial y} = 3x^2 y + g'(y) = 3 + 3x^2 y \Rightarrow g(y) = 3y$$

$$\Rightarrow z = f(x,y) = \frac{3x^2 y^2}{2} + 5x + 3y$$