EQUATIONS OF PLANES - ANSWERS

If possible, for each problem below find the equation for the plane containing the point $P=(1,2,3)$ and the vectors $\vec{u}$ and $\vec{v}$. Write your answer in the form $z=Ax+By+C$, if possible.

1. $\vec{u}=2\hat{i}+3\hat{j}$ and $\vec{v}=3\hat{i}-2\hat{j}$

$$\vec{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 0\\ 3 & -2 & 0\end{vmatrix}=(-4-9)\hat{k}=-13\hat{k}=0\hat{i}+0\hat{j}-13\hat{k}$$

$P=(1,2,3)$

$0(x-1)+0(y-2)-13(z-3)=0\Rightarrow -13z+39=0\Rightarrow z=3$

2. $\vec{u}=2\hat{i}+3\hat{j}$ and $\vec{v}=4\hat{i}+3\hat{j}$

$$\vec{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 0\\ 4 & 3 & 0\end{vmatrix}=(6-12)\hat{k}=-6\hat{k}=0\hat{i}+0\hat{j}-6\hat{k}$$

$P=(1,2,3)$

$0(x-1)+0(y-2)-6(z-3)=0\Rightarrow -6z+18=0\Rightarrow z=3$

3. $\vec{u}=\hat{i}+\hat{j}-5\hat{k}$ and $\vec{v}=2\hat{i}+2\hat{j}-4\hat{k}$

$$\vec{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -5\\ 2 & 2 & -4\end{vmatrix}=(-4+10)\hat{i}-(-4+10)\hat{j}+(2-2)\hat{k}=6\hat{i}-6\hat{j}+0\hat{k}$$

$P=(1,2,3)$

$6(x-1)-6(y-2)+0(z-3)=0\Rightarrow 6x-6y+6=0\Rightarrow x-y+1=0\Rightarrow y=x+1$

4. $\vec{u} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{v} = 3\hat{i} - 2\hat{j} + \hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & -2 & 1 \end{vmatrix} = (3+2)\hat{i} - (2-3)\hat{j} + (-4-9)\hat{k} = 5\hat{i} + \hat{j} - 13\hat{k}$$

$P = (1,2,3)$

$$5(x-1) + (y-2) - 13(z-3) = 0 \Rightarrow 5x + y - 13z + 32 = 0 \Rightarrow z = \frac{5}{13}x + \frac{1}{13}y + \frac{32}{13}$$

5. $\vec{u} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{v} = 2\hat{i} + 2\hat{j} - 10\hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 2 & 2 & -10 \end{vmatrix} = (-30-2)\hat{i} - (-20-2)\hat{j} + (4-6)\hat{k} = -32\hat{i} + 22\hat{j} - 2\hat{k}$$

$P = (1,2,3)$

$$-32(x-1) + 22(y-2) - 2(z-3) = 0 \Rightarrow -32x + 22y - 2z - 6 = 0 \Rightarrow z = -16x + 11y - 3$$