DIRECTIONAL DERIVATIVES - ANSWERS

For each of the following functions, find the directional derivative at the point $(1,1)$ in the direction $\vec{u} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$. If necessary, round to four decimal places.

$$D_{\vec{u}} f(1,1) = \nabla f(1,1) \bullet \vec{u}$$

1. $z = f(x, y) = x^3 y^2$

$$\nabla f = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = 3x^2y^2\,\hat{i} + 2x^3y\,\hat{j} = \left\langle 3x^2y^2, 2x^3y \right\rangle$$

$$\nabla f(1,1) = \langle 3, 2 \rangle$$

$$D_{\vec{u}} f(1,1) = \nabla f(1,1) \bullet \vec{u} = \langle 3, 2 \rangle \bullet \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}} \approx 3.5355$$

$$\max = \left\| \nabla f(1,1) \right\| = \left\| \langle 3, 2 \rangle \right\| = \sqrt{13} \approx 3.6056$$

$$\min = -\left\| \nabla f(1,1) \right\| \approx -3.6056$$

2. $z = f(x, y) = \sin(x^3 y^2)$

$$\nabla f = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \cos(x^3y^2)\cdot 3x^2y^2\,\hat{i} + \cos(x^3y^2)\cdot 2x^3y\,\hat{j}$$

$$= \left\langle 3\cos(x^3y^2)x^2y^2, 2\cos(x^3y^2)x^3y \right\rangle$$

$$\nabla f(1,1) = \langle 3\cos(1), 2\cos(1) \rangle$$

$$D_{\vec{u}} f(1,1) = \nabla f(1,1) \bullet \vec{u} = \langle 3\cos(1), 2\cos(1) \rangle \bullet \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \frac{3\cos(1)}{\sqrt{2}} + \frac{2\cos(1)}{\sqrt{2}} = \frac{5\cos(1)}{\sqrt{2}} \approx 1.9103$$

$$\max = \left\| \nabla f(1,1) \right\| = \left\| \langle 3\cos(1), 2\cos(1) \rangle \right\| \approx 1.9481$$

$$\min = -\left\| \nabla f(1,1) \right\| \approx -1.9481$$

3. $z = f(x,y) = \sqrt{x^3y^2}$

$$\nabla f = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \frac{1}{2\sqrt{x^3y^2}}\cdot 3x^2y^2\,\hat{i} + \frac{1}{2\sqrt{x^3y^2}}\cdot 2x^3y\,\hat{j}$$

$$= \left\langle 3\frac{1}{2\sqrt{x^3y^2}}x^2y^2, 2\frac{1}{2\sqrt{x^3y^2}}x^3y \right\rangle = \left\langle \frac{3x^2y^2}{2\sqrt{x^3y^2}}, \frac{x^3y}{\sqrt{x^3y^2}} \right\rangle$$

$$\nabla f(1,1) = \left\langle \frac{3}{2}, 1 \right\rangle$$

$$D_{\vec{u}} f(1,1) = \nabla f(1,1) \bullet \vec{u} = \left\langle \frac{3}{2}, 1 \right\rangle \bullet \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \frac{3}{2\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{5}{2\sqrt{2}} \approx 1.7678$$

$$\max = \|\nabla f(1,1)\| = \left\| \left\langle \frac{3}{2}, 1 \right\rangle \right\| = \sqrt{\frac{13}{4}} \approx 1.8028$$

$$\min = -\|\nabla f(1,1)\| \approx -1.8028$$

4. $z = f(x,y) = \sec(x^3y^2)$

$$\nabla f = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \sec(x^3y^2)\tan(x^3y^2)\cdot 3x^2y^2\,\hat{i} + \sec(x^3y^2)\tan(x^3y^2)\cdot 2x^3y\,\hat{j}$$

$$= \left\langle 3\sec(x^3y^2)\tan(x^3y^2)x^2y^2, 2\sec(x^3y^2)\tan(x^3y^2)x^3y \right\rangle$$

$$\nabla f(1,1) = \left\langle 3\sec(1)\tan(1), 2\sec(1)\tan(1) \right\rangle$$

$$D_{\vec{u}} f(1,1) = \nabla f(1,1) \bullet \vec{u} = \left\langle 3\sec(1)\tan(1), 2\sec(1)\tan(1) \right\rangle \bullet \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$= \frac{3\sec(1)\tan(1)}{\sqrt{2}} + \frac{2\sec(1)\tan(1)}{\sqrt{2}} = \frac{5\sec(1)\tan(1)}{\sqrt{2}} \approx 10.1911$$

5. $z = f(x,y) = \tan(x^3y^2)$

$$\nabla f = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \sec^2(x^3y^2)\cdot 3x^2y^2\,\hat{i} + \sec^2(x^3y^2)\cdot 2x^3y\,\hat{j}$$

$$= \left\langle 3\sec^2(x^3y^2)x^2y^2, 2\sec^2(x^3y^2)x^3y \right\rangle$$

$$\nabla f(1,1) = \left\langle 3\sec^2(1), 2\sec^2(1) \right\rangle$$

$$D_{\vec{u}} f(1,1) = \nabla f(1,1) \bullet \vec{u} = \left\langle 3\sec^2(1), 2\sec^2(1) \right\rangle \bullet \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$= \frac{3\sec^2(1)}{\sqrt{2}} + \frac{2\sec^2(1)}{\sqrt{2}} = \frac{5\sec^2(1)}{\sqrt{2}} \approx 12.1110$$