# CURL AND DIVERGENCE

The curl and divergence of a vector field $F = P(x,y,z)\hat{i} + Q(x,y,z)\hat{j} + R(x,y,z)\hat{k}$ are defined as follows:

$$curl\ of\ F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Q & R \end{vmatrix}\hat{i} - \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ P & R \end{vmatrix}\hat{j} + \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ P & Q \end{vmatrix}\hat{k}$$

$$= \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k}$$

Note that if ***F*** is a 2-dimensional vector field, then we just set *R=0* and $curl\ of\ F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k}$.

$$divergence\ of\ F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

If ***F*** is a 2-dimensional vector field, then this just reduces to $divergence\ of\ F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$

As we'll see later on, curl and divergence help measure, respectively, the tendency of a vector field to create circulation about a point or flux across a boundary. For each of the vector fields below, find the curl and divergence at the point (1,1) in problems 1-10, at (1,1,1) in problems 11-14, and at (0,0,0) in problem 15.

1. $F(x,y) = -y\hat{i} + x\hat{j}$
2. $F(x,y) = -y\hat{i} - x\hat{j}$
3. $F(x,y) = y\hat{i} - x\hat{j}$
4. $F(x,y) = \hat{i} - x\hat{j}$
5. $F(x,y) = x\hat{i} + y\hat{j}$
6. $F(x,y) = -x\hat{i} - y\hat{j}$
7. $F(x,y) = \hat{i} + (x+y)\hat{j}$
8. $F(x,y) = |x|\hat{i}$
9. $F(x,y) = |y|\hat{j}$
10. $F(x,y) = -y\hat{i} + (x+y)\hat{j}$
11. $F(x,y,z) = x\hat{i} + y\hat{j} + z\hat{k}$
12. $F(x,y,z) = xyz\hat{i} + xy^2\hat{j} + yz\hat{k}$
13. $F(x,y,z) = -x\hat{i} - y\hat{j} - z\hat{k}$
14. $F(x,y,z) = -x\hat{i} - y\hat{j} + z\hat{k}$
15. $F(x,y,z) = \cos(x)\hat{i} + \sin(x)\hat{j} + z\hat{k}$