CURL AND DIVERGENCE - ANSWERS

For each of the vector fields below, find the curl and divergence at the point (1,1) in problems 1-10, at (1,1,1) in problems 11-14, and at (0,0,0) in problem 15.

1. $F(x,y) = -y\hat{i} + x\hat{j}$

$$curl\,F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = (1-(-1))\hat{k} = 2\hat{k}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 + 0 = 0$$

2. $F(x,y) = -y\hat{i} - x\hat{j}$

$$curl\,F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = (-1-(-1))\hat{k} = 0\hat{k} = \vec{0}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 + 0 = 0$$

3. $F(x,y) = y\hat{i} - x\hat{j}$

$$curl\,F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = (-1-1)\hat{k} = -2\hat{k}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 + 0 = 0$$

4. $F(x,y) = \hat{i} - x\hat{j}$

$$curl\,F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = (-1-0)\hat{k} = -\hat{k}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 + 0 = 0$$

5. $F(x,y) = x\hat{i} + y\hat{j}$

$$curl\,F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = (0-0)\hat{k} = 0\hat{k} = \vec{0}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 1 + 1 = 2$$

6. $F(x,y) = -x\hat{i} - y\hat{j}$

$$curl\,F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = (0-0)\,\hat{k} = 0\hat{k} = \vec{0}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = -1 - 1 = -2$$

7. $F(x,y) = \hat{i} + (x+y)\,\hat{j}$

$$curl\,F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = (1-0)\,\hat{k} = \hat{k}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 + 1 = 1$$

8. $F(x,y) = |x|\,\hat{i} = x\hat{i}, \text{ for } x \geq 0$

$$curl\,F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = (0-0)\,\hat{k} = 0\hat{k} = \vec{0}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 1 + 0 = 1$$

9. $F(x,y) = |y|\,\hat{j} = y\hat{j}, \text{ for } y \geq 0$

$$curl\,F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = (0-0)\,\hat{k} = 0\hat{k} = \vec{0}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 + 1 = 1$$

10. $F(x,y) = -y\hat{i} + (x+y)\,\hat{j}$

$$curl\,F = \nabla \times F = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = (1-(-1))\,\hat{k} = 2\hat{k}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 + 1 = 1$$

11. $F(x,y,z) = x\hat{i} + y\hat{j} + z\hat{k}$

$$curl\,F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = 0\hat{i} + 0\,\hat{j} + 0\hat{k} = \vec{0}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 1 + 1 + 1 = 3$$

12. $F(x,y,z) = xyz\hat{i} + xy^2\hat{j} + yz\hat{k}$

$$curl\,F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = z\hat{i} + xy\hat{j} + (y^2 - xz)\hat{k}$$

$$\nabla \times F(1,1,1) = \hat{i} + \hat{j}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = yz + 2xy + y$$

$$\nabla \bullet F(1,1,1) = 1 + 2 + 1 = 4$$

13. $F(x,y,z) = -x\hat{i} - y\hat{j} - z\hat{k}$

$$curl\,F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = 0\hat{i} - 0\hat{j} + 0\hat{k} = \vec{0}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = -1 - 1 - 1 = -3$$

14. $F(x,y,z) = -x\hat{i} - y\hat{j} + z\hat{k}$

$$curl\,F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = 0\hat{i} - 0\hat{j} + 0\hat{k} = \vec{0}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = -1 - 1 + 1 = -1$$

15. $F(x,y,z) = \cos(x)\hat{i} + \sin(x)\hat{j} + z\hat{k}$

$$curl\,F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} = 0\hat{i} - 0\hat{j} + \cos x\,\hat{k} = \cos x\,\hat{k}$$

$$\nabla \times F(0,0,0) = \hat{k}$$

$$div\,F = \nabla \bullet F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = -\sin x + 0 + 1$$

$$\nabla \bullet F(0,0,0) = 0 + 0 + 1 = 1$$