COMPONENTS AND PROJECTIONS - ANSWERS

In each of the problems below, you are given a force vector $\vec{F}$ and a distance vector $\vec{d}$. Suppose the magnitude of $\vec{F}$ corresponds to the number of pounds of force and the magnitude of $\vec{d}$ corresponds to a distance in feet that an object is moved by the force. For each of the problems below find $comp_{\vec{d}}\vec{F}$, $proj_{\vec{d}}\vec{F}$, and the work done by $\vec{F}$ in moving the object the length of $\vec{d}$. Give exact answers, and on the latter, use units of *foot-pounds*.

1. $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{d} = 2\hat{i} + 2\hat{j} + 5\hat{k}$

   work $= 21$ foot-pounds

   $comp_{\vec{d}}\vec{F} = \dfrac{21}{\sqrt{33}} = \dfrac{7\sqrt{33}}{11}$

   $proj_{\vec{d}}\vec{F} = \dfrac{21}{33}\vec{d} = \dfrac{7}{11}\vec{d} = \dfrac{14}{11}\hat{i} + \dfrac{14}{11}\hat{j} + \dfrac{35}{11}\hat{k}$

2. $\vec{F} = 3\hat{i} + \hat{j} + 4\hat{k}$, $\vec{d} = 8\hat{i} + 2\hat{j} + 6\hat{k}$

   work $= 50$ foot-pounds

   $comp_{\vec{d}}\vec{F} = \dfrac{25}{\sqrt{26}} = \dfrac{25\sqrt{26}}{26}$

   $proj_{\vec{d}}\vec{F} = \dfrac{25}{52}\vec{d} = \dfrac{50}{13}\hat{i} + \dfrac{25}{26}\hat{j} + \dfrac{75}{26}\hat{k}$

3. $\vec{F} = 3\hat{i} + 2\hat{j}$, $\vec{d} = 10\hat{i}$

   work $= 30$ foot-pounds

   $comp_{\vec{d}}\vec{F} = \dfrac{30}{10} = 3$

   $proj_{\vec{d}}\vec{F} = \dfrac{30}{100}\vec{d} = 3\hat{i}$

4. $\vec{F} = \hat{i} + \hat{j}$, $\vec{d} = 5\hat{i} + \hat{j}$

   work $= 6$ foot-pounds

   $comp_{\vec{d}}\vec{F} = \dfrac{6}{\sqrt{26}} = \dfrac{3\sqrt{26}}{13}$

   $proj_{\vec{d}}\vec{F} = \dfrac{6}{26}\vec{d} = \dfrac{3}{13}\vec{d} = \dfrac{15}{13}\hat{i} + \dfrac{3}{13}\hat{j}$

5. $\vec{F} = 2\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{d} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

   work $= 12$ foot-pounds

   $comp_{\vec{d}}\vec{F} = \dfrac{6}{\sqrt{3}} = 2\sqrt{3}$

   $proj_{\vec{d}}\vec{F} = \left(\dfrac{12}{12}\right)\vec{d} = 2\hat{i} + 2\hat{j} + 2\hat{k}$