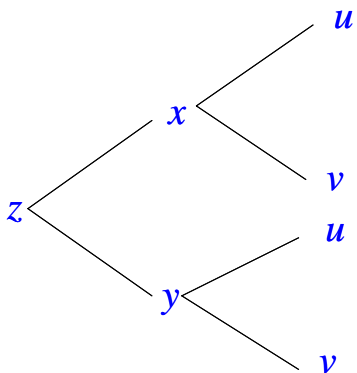


## CHAIN RULE DIAGRAMS – ANSWERS

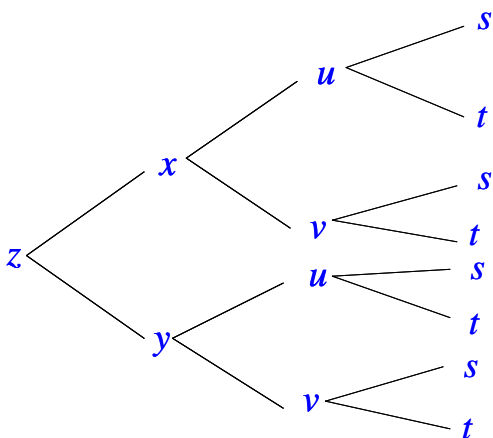
Construct a tree diagram and a chain rule formula for each of the indicated derivatives.

1.  $z = f(x, y), x = x(u, v), y = y(u, v), \frac{\partial z}{\partial u} = ?$



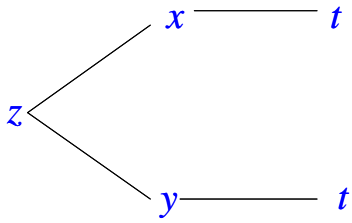
$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u}$$

2.  $z = f(x, y), x = x(u, v), y = y(u, v), u = u(s, t), v = v(s, t), \frac{\partial z}{\partial s} = ?$



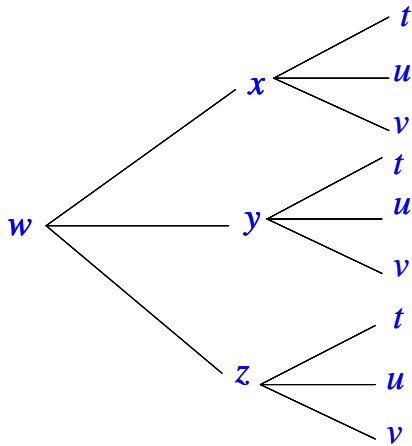
$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} \frac{\partial u}{\partial s} + \frac{\partial z}{\partial x} \frac{\partial x}{\partial v} \frac{\partial v}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u} \frac{\partial u}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial v} \frac{\partial v}{\partial s}$$

3.  $z = f(x, y), x = x(t), y = y(t), \frac{dz}{dt} = ?$



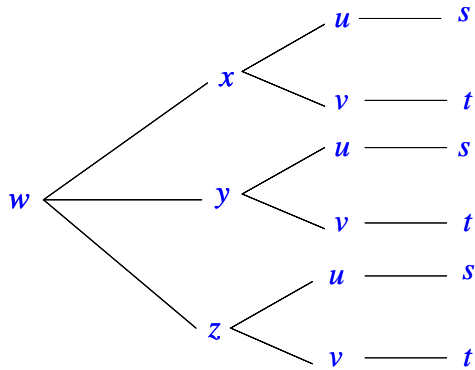
$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$$

4.  $w = f(x, y, z), x = x(t, u, v), y = y(t, u, v), z = z(t, u, v), \frac{\partial w}{\partial t} = ?$



$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial t}$$

5.  $w = f(x, y, z)$ ,  $x = x(u, v)$ ,  $y = y(u, v)$ ,  $z = z(u, v)$ ,  $u = u(s)$ ,  $v = v(t)$ ,  $\frac{\partial w}{\partial t} = ?$



$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial v} \frac{dv}{dt} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial v} \frac{dv}{dt} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial v} \frac{dv}{dt}$$