CHAIN RULE - ANSWERS

If $x = t^3$ and $y = \sin t$, use the chain rule to find $\frac{dz}{dt}$. Show your work!

$\frac{dx}{dt} = 3t^2$, $\frac{dy}{dt} = \cos t$

1. $z = f(x, y) = x^3 y^2$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = 3x^2 y^2 \cdot 3t^2 + 2x^3 y \cdot \cos t$$
$$= 3(t^3)^2 \sin^2 t \cdot 3t^2 + 2(t^3)^3 \sin t \cdot \cos t$$
$$= 9t^8 \sin^2 t + 2t^9 \sin t \cos t$$

2. $z = f(x, y) = \sin(x^3 y^2)$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = \cos(x^3 y^2) \cdot 3x^2 y^2 \cdot 3t^2 + \cos(x^3 y^2) \cdot 2x^3 y \cdot \cos t$$
$$= \cos((t^3)^3 \sin^2 t) \cdot 3(t^3)^2 \sin^2 t \cdot 3t^2 + \cos((t^3)^3 \sin^2 t) \cdot 2(t^3)^3 \sin t \cdot \cos t$$
$$= \cos\left(t^9 \sin^2 t\right)\left[9t^8 \sin^2 t + 2t^9 \sin t \cos t\right]$$

3. $z = f(x, y) = \sqrt{x^3 y^2}$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = \frac{1}{2\sqrt{x^3 y^2}} \cdot 3x^2 y^2 \cdot 3t^2 + \frac{1}{2\sqrt{x^3 y^2}} \cdot 2x^3 y \cdot \cos t$$
$$= \frac{1}{2\sqrt{(t^3)^3 \sin^2 t}} \cdot 3(t^3)^2 \sin^2 t \cdot 3t^2 + \frac{1}{2\sqrt{(t^3)^3 \sin^2 t}} \cdot 2(t^3)^3 \sin t \cdot \cos t$$
$$= \frac{1}{2\sqrt{t^9 \sin^2 t}}\left[9t^8 \sin^2 t + 2t^9 \sin t \cos t\right]$$

4. $z = f(x,y) = \sec(x^3 y^2)$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = \sec(x^3 y^2)\tan(x^3 y^2)\cdot 3x^2 y^2 \cdot 3t^2 + \sec(x^3 y^2)\tan(x^3 y^2)\cdot 2x^3 y \cdot \cos t$$

$$= \sec((t^3)^3 \sin^2 t)\tan((t^3)^3 \sin^2 t)\cdot 3(t^3)^2 \sin^2 t \cdot 3t^2$$

$$+ \sec((t^3)^3 \sin^2 t)\tan((t^3)^3 \sin^2 t)\cdot 2(t^3)^3 \sin t \cdot \cos t$$

$$= \sec\left(t^9 \sin^2 t\right)\tan\left(t^9 \sin^2 t\right)\left[9t^8 \sin^2 t + 2t^9 \sin t \cos t\right]$$

5. $z = f(x,y) = \tan(x^3 y^2)$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = \sec^2(x^3 y^2)\cdot 3x^2 y^2 \cdot 3t^2 + \sec^2(x^3 y^2)\cdot 2x^3 y \cdot \cos t$$

$$= \sec^2((t^3)^3 \sin^2 t)\cdot 3(t^3)^2 \sin^2 t \cdot 3t^2 + \sec^2((t^3)^3 \sin^2 t)\cdot 2(t^3)^3 \sin t \cdot \cos t$$

$$= \sec^2\left(t^9 \sin^2 t\right)\left[9t^8 \sin^2 t + 2t^9 \sin t \cos t\right]$$