AREA OF A PARALLELOGRAM - ANSWERS

Find the area of the parallelogram determined by the following vectors.

1. $\vec{u} = 2\hat{i} + 3\hat{j}$ and $\vec{v} = 3\hat{i} - 2\hat{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 3 & -2 & 0 \end{vmatrix} = (-4 - 9)\hat{k} = -13\hat{k}$$

$\|\vec{u} \times \vec{v}\| = 13$

2. $\vec{u} = 2\hat{i} + 3\hat{j}$ and $\vec{v} = 4\hat{i} + 6\hat{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 4 & 6 & 0 \end{vmatrix} = 0\hat{k} = \vec{0}$$

$\|\vec{u} \times \vec{v}\| = 0$

Zero area, the vectors are parallel

3. $\vec{u} = 2\hat{i} + 3\hat{j}$ and $\vec{v} = -6\hat{i} - 9\hat{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ -6 & -9 & 0 \end{vmatrix} = (-18 + 18)\hat{k} = 0\hat{k} = \vec{0}$$

$\|\vec{u} \times \vec{v}\| = 0$

Zero area, the vectors are parallel

4. $\vec{u} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{v} = 3\hat{i} - 2\hat{j} + \hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & -2 & 1 \end{vmatrix} = (3 + 2)\hat{i} - (2 - 3)\hat{j} + (-4 - 9)\hat{k} = 5\hat{i} + \hat{j} - 13\hat{k}$$

$\|\vec{u} \times \vec{v}\| = \sqrt{25 + 1 + 169} = \sqrt{195} \approx 13.96$

5. $\vec{u} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{v} = 2\hat{i} + 2\hat{j} - 10\hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 2 & 2 & -10 \end{vmatrix} = (-30 - 2)\hat{i} - (-20 - 2)\hat{j} + (4 - 6)\hat{k} = -32\hat{i} + 22\hat{j} - 2\hat{k}$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{1024 + 484 + 4} = \sqrt{1512} = 6\sqrt{42} \approx 38.88$$