STOKES' THEOREM IN THREE DIMENSIONS

In each problem below you are given a surface *S*, defined by $z = f(x, y)$, over a region *R*, defined by the given limits on *x* and *y*. Let C_R be the boundary of the region *R*, oriented counterclockwise, and let *C* be the corresponding boundary on the surface *S*, also oriented counterclockwise. Then if *F* is a vector field and *N* is the upward pointing unit normal vector for the surface *S*, use the higher dimensional version of Stokes' Theorem, $\int_C \vec{F} \bullet d\vec{r} = \iint_S \left(curl\, F \bullet N \right) dS$, to measure the circulation around the curve *C* that is caused by the vector field *F*.

1. $S: z = -x^2 - y^2 + 4$
   $R: 0 \le x \le 1,\ 0 \le y \le 2$
   $F = z\hat{i} + x\hat{j} + y\hat{k}$

2. $S: z = y$
   $R: 0 \le x \le 1,\ 0 \le y \le 1 - x$
   $F = -3y^2\hat{i} + 4z\hat{j} + 6x\hat{k}$

3. $S: z = x^2 - y^2$
   $R: -1 \le x \le 1,\ -\sqrt{1 - x^2} \le y \le \sqrt{1 - x^2}$
   $F = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$