

## DIVERGENCE THEOREM IN THREE DIMENSIONS

In each problem below you are given a volume  $V$  bounded by a surface  $S$  along with a vector field  $F$ . If  $N$  is an outward pointing unit normal vector, then the outward flux across the surface created by the vector field  $F$  is, by the Divergence (Gauss') Theorem, equal to  $\iint_S \vec{F} \cdot N dS = \iiint_V \operatorname{div} \vec{F} dV = \iiint_V \nabla \cdot F dV$ . Evaluate this integral for each problem below

1.  $V$  is the solid ball with surface  $S$  defined by  $x^2 + y^2 + z^2 = 1$

$$F = x\hat{i} + y\hat{j} + z\hat{k}$$

2.  $V$  is the cube defined by  $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$ , and  $S$  is the surface of the cube

$$F = x^2\hat{i} + y\hat{j} + z\hat{k}$$

$V$  is the cylinder defined by  $-1 \leq x \leq 1, -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}, 0 \leq z \leq 4$ ,

3. and  $S$  is the surface of the cylinder

$$F = y\hat{i} - x\hat{j}$$