Algebraic Models for Constructivist Theories of Perception

by

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In constructivist approaches to the perception of reality our perceived world is seen entirely as a creation of the mind that is constructed piece by piece, layer by layer. In fact, whether an outside world exists at all is sometimes seen as both an unanswerable and an irrelevant question. All we know for sure is what we see[1], and what we see takes place entirely in the mind. In this paper we will propose some simple algebraic models for constructivist theories of cognition, and while in many respects these models are elementary, they, nonetheless, draw upon the framework and terminology of both group theory and algebraic topology. Consequently, some knowledge of both of these areas is desired. We begin with the notion of a **0-simplex** as the basis for an individual's constructed world.

The 0-Simplex

At the beginning of life, we have what we might characterize as primary perceptions. Pure, raw, unprocessed awareness. A dot here, a splash of color there. No organization, yet, into things such as a chair, house, car, etc. We will refer to these initial perceptions as **0-simplices**. We will also assume that at any given moment, perception is based upon a finite, rather than an infinite, number of 0-simplices.

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[1] Throughout we will frequently use "seeing" in the most general manner possible to refer to any type of sensory perception.

The Perceptual Groups

We can associate several mathematical *groups* with any perceived collection of 0-simplices. We can, for example, let the 0-simplices be generators for *free abelian groups*. In this setup, the positive integer coefficient given to a particular 0-simplex represents the amount of attention focused on that particular element. Similarly, a coefficient of zero would indicate that the particular 0-simplex is not part of the current perception, and a negative coefficient could be interpreted in terms of distance from conscious awareness. Thus, if we gave a 0-simplex a coefficient of -10, in lieu of zero, then that could mean that not only is the given simplex not part of the current perception, it is also quite likely not to be part of any other perception in the near future. In this model, simplices with a coefficient of zero would be just on the border between awareness and nonawareness. This corresponds well to what we experience in reality. For instance, consider Einstein's theory of special relativity which may be derived using only high school algebra coupled with some very clever thinking. In the early 1900s, in Einstein's case, the discovery of this theory was right on the cusp of his awareness. Similarly, anyone else with the same knowledge of basic algebra and physics could have made the same discovery. However, for a person lacking the requisite knowledge of math and physics, the theory would not only not be a part of conscious awareness, it would, in a very real sense, be very far away from awareness. Thus, just as we can assign positive integers to represent the amount of attention focused on a given perception, we can also use negative coefficients to indicate the relative distance from consciousness. To give another illustration, suppose that a person's entire perception consisted only of the colors red, green, and blue and nothing else. In this case, a person's current perception could be indicated by a simple 3-tuple with integer coefficients showing the magnitude of the perception of each color, and algebraically, the resulting group is isomorphic to $\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$. Consequently, the 3-tuple (1-red, 1-green, 1-blue) would represent perception of red, green, and blue at equal strengths. On the other hand, (2-red, 10-green, 5-blue) would mean that, while still seeing all three colors, the person is focused most on green, next on blue, and least on red. And finally, the perception (2-red, 0-green, -5-blue) would be interpreted as meaning that we are perceiving red at a strength symbolized by 2, and we are not perceiving green nor blue at all. Additionally, green is on the periphery of consciousness while blue, with its negative coefficient, is further removed.

We can generate another group theoretic model by letting each 0-simplex be the generator of a *group* isomorphic to $\mathbb{Z}_n$, for some integer *n*, and then take the *direct sum* of these *groups*. In this model, the value of *n* for any particular 0-simplex will place an upper bound on the strength of any perception of that simplex, and at the same time negative coefficients are eliminated. For example, let's continue with our model of (red, green, blue), but this time let's let the resulting group be isomorphic to $\mathbb{Z}_5 \oplus \mathbb{Z}_{10} \oplus \mathbb{Z}_3$. This creates a group where the amount of attention that can be invested in green is greater than that of red which, in turn, is greater than that of blue, and such a group seems to correspond better to a world where I, for example, am able to typically focus in much greater detail on mathematics than, say, baseball.

A third group model can be created as follows. If σ is a 0-simplex, we can let $\mathbb{Z}$ represent the *free group* generated by σ. Next, map $\mathbb{Z}$ onto $5\mathbb{Z}$ (for example), and then map $5\mathbb{Z}$ onto a group of order 2 (i.e. $5\mathbb{Z}/10\mathbb{Z}$). The end result is a group of order 2, but one in which the coefficient has been adjusted so that the elements of the group may be thought of as 0σ and 5σ. <u>If we follow this procedure with each 0-simplex perceived, then we can think of the resulting coefficient as representing the amount of attention paid to that particular 0-simplex</u>. This results in a group that represents an accurate snapshot of the state of our current perception. Or, to make things even simpler, we could just assign a coefficient of either 0 or 1 to each object/simplex in order to indicate either the object's absence or presence. This results in a perceptual group that can be characterized by a direct sum of copies of $\mathbb{Z}_2$.

If we are observing some scene of 0-simplices, then the richest algebraic formulation would be to follow our initial *free abelian group* construction for each object in that scene. However, to keep things simpler at this point, we will utilize our latter scheme involving a direct sum of copies of $\mathbb{Z}_2$, and we will refer to the resulting group as the **0-dimensional perceptual group** for that particular scene. Formulations involving the other group models can then be made as desired, *mutatis mutandis*. Furthermore, in all of these group models it will be convenient to associate the identity element with the observer since in the dichotomy of the observer and the observed, that which does the observing is never a direct object of perception. Hence, it corresponds to the element of the perceptual group that has all of its coordinates equal to zero.

Higher Dimensional Simplices

Just as points come together to form lines, more general perceptions come together to form objects, and sounds come together in our minds to form words and sentences. In such a manner is our reality created. Thus, we think of 0-simplices coming together to form what we will call 1-simplices, and 1-simplices coming together to form 2-simplices, and so on and so on.

When we learn to see the forest instead of the trees, that which allows us to see the individual trees has to momentarily disappear. Similarly, when we transition from the set of all integers, $\{\ldots -3,-2,-1,0,1,2,3\ldots\}$, to only the set of even or odd, $\{\text{even, odd}\}$, we are making the differences between integers that are multiples of 2 momentarily disappear. In the language of group theory, we say that by grouping the even and odd integers into two equivalence classes, we create a quotient group in which no distinction is made between the even integers and no distinction is made between the various odd integers. The result can be characterized as a homomorphism from $\mathbb{Z} \to \mathbb{Z}_2$, and we also frequently say that the differences between the various even integers (and likewise for the odd integers) have been factored out. In a similar manner, we will say that 1-simplices are formed from 0-simplices when something non-trivial has been factored out. Something that causes us to see a particular set of 0-simplices as separate or unrelated has to be made to disappear in order to create a higher dimensional structure. Thus, we will understand a 1-simplex as a non-trivial quotient[2] of a 0-dimensional perceptual group. The kernel of the morphism is whatever has to be factored out to make the resulting objects appear as they do, and the result is a new perceptual group of dimension 1. **In general, we define a perceptual group of dimension n > 0 to be any nontrivial quotient of a perceptual group of dimension n – 1, and we define a simplex of dimension n > 0 to be any nontrivial subgroup of a perceptual group of dimension n**. Additionally, because an *n dimensional perceptual group* is a quotient of an *n*-1 *dimensional perceptual group*, we will often refer to the identity element of the quotient group as its *kernel*. Furthermore, since an identity element occurs in all *n dimensional perceptual groups*, it may be more convenient not to assign any particular dimension to the identity or kernel. Also, we may associate a kernel with the object perceived in the following way. Suppose what we have before us are only three trees, and the corresponding perceptual group is isomorphic to $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$ where we might depict the elements of his group as

$$\left\{ \begin{matrix} (0,0,0) \\ (tree1,0,0) \\ (0,tree2,0) \\ (0,0,tree3) \\ (tree1,tree2,0) \\ (0,tree2,tree3) \\ (tree1,0,tree3) \\ (tree1,tree2,tree3) \end{matrix} \right\}.$$

In this context, we might think of the element $(tree1,tree2,tree3)$ as representing the forest instead of the trees, and when we form the normal subgroup $\left\{ \begin{matrix} (0,0,0) \\ (tree1,tree2,tree3) \end{matrix} \right\}$, we might think of the equivalence class in the resulting quotient group as representing an

[2] By non-trivial quotient group in this context, we mean that the kernel is neither the identity nor the entire group.

identification of the observer, $(0,0,0)$, with the forest, $(tree1, tree2, tree3)$. Thus, in the resulting quotient group the "forest" is united with consciousness and that is the object we perceive, and this particular way of interpreting things suggests that perhaps the only kernels we should be interested in are those of order 2 since, in that case, one element will correspond to the observer and the other element to the items that are being combined in order to form our object. Additionally, it is worth noting that this is the way in which many eastern philosophies analyze perception, i.e. in terms of consciousness combining itself with a particular object. These philosophies go on to teach us that it is this identification that is the cause of much of our sorrow, and that to alleviate our suffering in this world we need to break that identification between consciousness and the object, between the observer and the observed.

In the scenario just envisioned, we wind up with a quotient group that contains just four elements or equivalence classes. Namely, $\left\{\begin{matrix}(0,0,0)\\(tree1,tree2,tree3)\end{matrix}\right\}$, $\left\{\begin{matrix}(tree1,0,0)\\(0,tree2,tree3)\end{matrix}\right\}$, $\left\{\begin{matrix}(0,tree2,0)\\(tree1,0,tree3)\end{matrix}\right\}$, and $\left\{\begin{matrix}(0,0,tree3)\\(tree1,tree2,0)\end{matrix}\right\}$. In this situation, not only can we say that we are seeing the forest instead of the trees, we can also say that an equivalence class like $\left\{\begin{matrix}(tree1,0,0)\\(0,tree2,tree3)\end{matrix}\right\}$ tells us that we can no longer separate *tree*1 from its complement, *tree*2 and *tree*3. And now for the interesting part. <u>The four-element group given above is what we'll call the expanded version of our simplex or, more simply, the expanded simplex. It shows all the component parts. However, in practice, we often just refer to the final result as "forest" without any additional, non-trivial components shown. Thus, when we</u>

add this simplex to a perceptual group, we will often treat it as simply the two-element group $\begin{Bmatrix} 0 \\ forest \end{Bmatrix}$ or, if required, as just $\{forest\}$. We will call both of these forms the short simplex. This practice is justified because, for example, in the real world when we refer to something such as our car, we don't usually think about any other components or parts. We just think about the concept "car." In fact, many of us will often experience difficulty when it comes to expanding a neat, tidy little concept like "car" back into its component parts.

Knowledge

The pulling together of several bits of information or experience in order to arrive at some new concept is often referred to as wisdom, understanding, and knowledge. For example, our experiences of 1-element sets all come together to form our understanding of the number 1. Similarly, our experiences with people in all walks of life come together to form our understanding of relationships. Whenever there is a coming together, there is also a factoring out of that which previously kept the elements separate and apart. Consequently, we can now identify the process of **wisdom/understanding/knowledge** as a quotient group. Whenever we have an epiphany or an "aha" moment, our brains suddenly combine several component parts together to form a new understanding, a new way of looking at things. Immediately after our brief moment of right brain wisdom or enlightenment, the left hemisphere of the brain begins to wrap and mold the new found wisdom into a more structured form of understanding, and then the coordinated functioning of the two brain hemispheres results in a new level of knowledge as a result of a radical reorganization that has factored out portions of the old world view in order to create the new. Accordingly, in our model of perception, <u>insight and intelligence are more a matter of forming quotient groups than they are of algorithms</u>.

A Few More Details

When we look about a room, we see a myriad of objects. These objects can be used as generators for a corresponding *perceptual group* that this time may include simplices of different dimensions, such as a chair and a leg of the chair. Specifically, this time we want to consider a group that is isomorphic to a finite number of copies of $\mathbb{Z}_2$ and that is constructed without regard to the dimension of the object. In other words, every time we construct a new simplex, we will add it as a *short simplex* to our general *perceptual group* which now contains all the objects we have constructed, regardless of dimension. Additionally, we can also give a revised definition for a simplex of dimension *n*. Instead of requiring it to be formulated from simplices of dimension *n*-1, we will now require that only one of its components be of dimension *n*-1 and then the others can be of even lesser dimension. Since we are constructing new models here, we are open to the possibility that some characteristics of a complex object, such as color, may be of a far lower dimension with regard to the concepts we have previously discussed.

There may also be other subsets of this perceptual group that could also be used as generators, but those elements that correspond to the individually perceived objects are obviously the preferred generators. For example, it is simpler to talk about the reality generated by the two element set $\{\text{table, chair}\}$ rather than resorting to a more complex set of generators such as $\{\text{table \& chair, chair}\}$. To give a more abstract example, what we just stated is analogous to saying that $\{(1,0),(0,1)\}$ is, in some sense, a more natural set of generators for $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ than $\{(1,1),(0,1)\}$. Also, along these same lines, one may want to call any nontrivial element of this group a simplex, but usually what we have in mind when we talk about a simplex is one of the individually perceived objects. Furthermore, if A & B represent two distinct objects/simplices that we see, then we may interpret the element A + B as meaning, "I see A and I see B." Additionally, when simplices are added in this manner, the resulting sum, in algebraic topology, is called a *chain*, and an arbitrary chain is frequently denoted by the symbol #. For instance, if σ_1, σ_2, and σ_3 are simplices, then a chain might look like $\# = 2\sigma_1 + 3\sigma_2 + \sigma_3$.

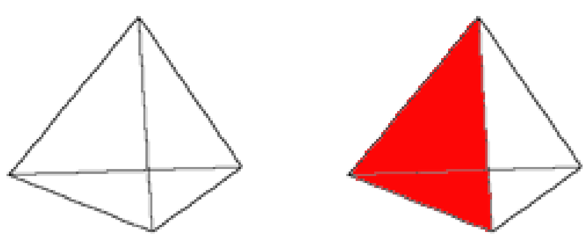
Something from Something

When we form a quotient structure, we are combining known elements together to form a new reality. This may also be referred to as *something from nothing creation* since it is that insight to combine certain elements into a whole that often seems to appear out of sheer nothingness. However, we can also alter our world without creating new objects by merely rearranging the old objects. In particular, we can imagine our *perceptual group* as not only containing a variety of objects, but also information on how those objects are to be arranged. If we create a different arrangement of those objects, then the result is what is often referred to as *something from something creation*. In this case, no new objects have been formed. Only the ordering of the old objects has been altered, or as I say, the only difference between a messy room and a clean room is the how the objects are arranged. And these are classically the two types of creation that one may engage in, *something from nothing creation* and *something from something creation*. The former refers to quotient structures, and the latter refers to permutations.

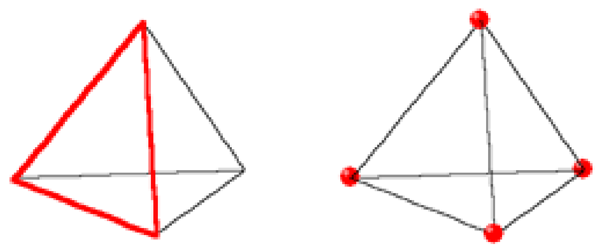
Faces

If σ is an *expanded simplex* in an *n dimensional perceptual group G*, then we will call any nontrivial element of σ an **n dimensional face** of σ. If H is a minimal group containing all the required components that must be brought together to form σ and of which σ is the respective quotient, then any nontrivial element of H, expressed as a *short simplex*, is an **n-1 dimensional face** of G. Faces of dimension $n\text{-}k \geq 0$ are defined similarly, and once again the identity is excluded from this dimensional definition.

The motivation for the definition of a face comes from elementary algebraic topology. For example, consider the 3-dimensional pyramid structure below. Any of the triangular sides of the pyramid is thought of as a 2-dimensional face.



Similarly, each line segment in the pyramid is a 1-dimensional face, and each vertex is a 0-dimensional face.



The geometric basis for algebraic topology is a spatial continuum that is infinitely divisible. However, our model is discrete instead of continuous, and that is the cause of some of the differences that occur in our group theoretical formulation.

At this point we might digress to note that there are some critical differences between the world as it is and the standard mathematical models for reality, and that is another reason why we are developing a discrete model instead of a continuous one. In particular, space is not infinitely divisible in the manner that the Cartesian plane or the real number line are. According to quantum physics, there can be no lengths smaller than the Planck length of 1.616×10^{-33} cm. When we observe space, it is characterized by discreteness rather than continuity. Furthermore, we generally do not find open sets in the real world. For example, when we break a pencil in two, we don't get one piece which is closed and another that is half-open. Nor can we cut an open circle out of a piece of cloth. Instead, everything we encounter in nature has a boundary. Thus, mathematical models based on discreteness have certain advantages over the usual continuous models. And that is why our model of perception is a group theoretical one in which objects are represented by equivalence classes that don't overlap. Other, non-discrete models can be good approximations for reality, but our errors will eventually catch up with us. When we observe the world, the world divides itself up into discrete particles, and, according to quantum physics, it becomes a continuous wave only when we are not looking at it[3]. Finally, for good measure, we note that neither do smooth curves appear to exist in nature. Instead, everything seems to be fractal and differentiable nowhere. Thus, even calculus is suspect.

The same object (i.e. simplex) may exist with respect to different sets of equivalence classes in the quotient groups that define different perceptual groups. For example, consider the direct sum $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$ that consists of the elements $\left\{ \begin{matrix} (0,0,0) \\ (horse,0,0) \\ (0,rider,0) \\ (0,0,chair) \\ (horse,rider,0) \\ (horse,0,chair) \\ (0,rider,chair) \\ (horse,rider,chair) \end{matrix} \right\}$.

Here we can think of the first component, or first factor $\mathbb{Z}_2$, as representing the horse, the second component/factor as representing the rider, and the third as representing the chair. However, if we wish to see the horse and the rider as one, then we must factor out the subgroup $\left\{ \begin{matrix} (0,0,0) \\ (horse,rider,0) \end{matrix} \right\}$. When we do this then on the one hand, we are only altering the first two components in our direct sum and essentially leaving the third component untouched. But on the other hand, we can also say that in our original group the chair

[3] "We have been taught that all this differentiation of the Divine Personality is from our side and relative to our knowledge, and that, above, all is one, all is set in one balance, unvarying and eternal, as it is written: 'I the Lord change not' (Malachi 3:6)." (Zohar II:176a)

appears as a 1-element coset, $\{(0,0,chair)\}$, but in the quotient group it now appears as a 2-element coset, $\left\{\begin{matrix}(0,0,chair)\\(horse,rider,chair)\end{matrix}\right\}$. When looked at from the perspective of these equivalence classes, we might want to say that the "chair" has different dimensions depending upon what quotient group it appears in. Nevertheless, we can still say that there is a unique set of elements that our mind has brought together to form that object, the chair, and hence, also a unique dimension. Thus, using our simplest group model, we may associate with each object (or simplex) a unique group that consists, as a quotient group, of all the elements that we have brought together in our mind to form the object and the corresponding kernel of elements that we have factored out.

As another example, let us consider a table. If we see the table in two different settings such as at the store and then at our home, then the same object is seen with respect to two different perceptual groups. The two perceptual groups, expressed as quotient groups, have different elements and kernels, and thus, different sets of equivalence classes. However, the table itself considered aside from the particular perceptual group within which it appears is always the same. That is to say, the elements such as top, legs, and so forth that must be brought together to form the table are the same regardless of the perceptual group in which the table appears. It is these elements that give us a unique way within our mind of representing the table as a quotient group of dimension *n*, and those items such as the top of the table, the legs, and the color of the table represent lesser dimensional faces of the table. Alternatively, remember that we may choose to use the models $\mathbb{Z}$ or $\mathbb{Z}_n$ in lieu of $\mathbb{Z}_2$. However, at this point, $\mathbb{Z}_2$ is simply the easiest group to describe.

Another lesson that may be derived from the above examples is that every time we combine elements together, every time we create a quotient structure, whether it be through intellectual insight or through two people becoming as one, our whole world changes. Even though a chair or a table will still appear the same in each reality, we also note that all the equivalence classes have, nonetheless, changed. Changing one part of our reality also changes the whole.

Orientations & Boundaries

By an **orientation** for an *n dimensional simplex*, *perceptual group*, or *subgroup* thereof, we mean an *n*-1 *dimensional group* plus the kernel for the desired quotient group that represents the *n-dimensional simplex*. In particular, we should choose the smallest possible *n*-1 *dimensional group*. For example, consider the group $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$, and suppose the n-dimensional object we are creating has as its kernel the subgroup generated by $(1,1,0)$. Since the third copy of $\mathbb{Z}_2$ is essentially uninvolved in this process, it is simpler to think of the orientation of our object or simplex as consisting merely of $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ and a kernel generated by $(1,1)$.

We also refer to an orientation as a **boundary** because it defines the resulting *n* dimensional simplex. We also note that each object perceived has a unique boundary in the sense that what we have factored out in our minds to create that object is unique. Simplices of dimension 0 have no boundary or orientation. And why do we define orientation in this manner? Because in ordinary language an orientation for an object is simply <u>any</u> characteristic of that object. Up, down, red, blue, or whatever. Likewise, every n-1 dimensional face of object can be thought of as defining a particular characteristic or orientation for that object. For example, if a table has three legs instead of two, or if a table is painted red, those characteristics of the component parts determine the orientation of the table. As a result, we find it preferable to essentially define the orientation of an n simplex in terms of the n-1 dimensional faces of that simplex. Similarly, we often think of something as being defined by its boundaries, and for that reason, we also refer to the orientation of an object as its boundary. Furthermore, when we take the smallest possible group needed to define an object, then the group, kernel and object may simultaneously be represented by a short exact sequence such as the following.

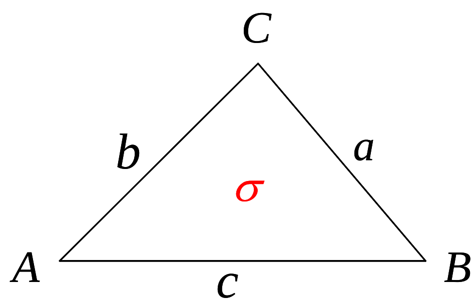
$$0 \longrightarrow H \longrightarrow G \longrightarrow G/H \longrightarrow 0$$

In this setup, we could also say that there is a certain duality involved. In other words, if we know H and G, then we certainly know G/H, and if we know the cosets in G/H, then we likewise know H and G.

Cycles

Traditionally, a boundary operator ∂ is defined in algebraic topology, and $*$ is called a cycle if $\partial(*)=0$. Furthermore, $*$ is called a bounding cycle if there exists a chain, $\#$, such that $\partial(\#)=*$. Such a formulation is not entirely suitable, however, for our approach.

For example, consider the triangle σ below, and recall that in Poincare's original formulation of algebraic topology, each element of a chain group had order 2.



Thus, he would have $\partial(\sigma) = a + b + c$, and

$$\begin{aligned}\partial(\partial(\sigma)) &= \partial(a) + \partial(b) + \partial(c) = (B + C) + (A + C) + (A + B) \\ &= (A + A) + (B + B) = (C + C) = 0 + 0 + 0 = 0.\end{aligned}$$

The property that $(\partial \circ \partial)(\sigma) = 0$ is dependent on the fact that the lines a, b, & c overlap. That is, a & b share C, a & c share B, and b & c share A. However, since in our formulation an *n*-simplex is always a set of equivalence classes forming a quotient group, there is never any overlap among its *n*-1 faces. Hence, we have to define cycles and bounding cycles in a different manner. Thus, by an **n-cycle** we mean any proper subgroup of an *n-dimensional perceptual group* since this defines a quotient group of dimension *n* + 1, and if this factor group is one that an individual has previously constructed in their mind, then we call that cycle a **bounding n-cycle**. To provide a real world context for this, consider once again a table and its component parts. The component parts have the potential to come together in our minds to create the perception of the table, and if we have, indeed, previously constructed the table in our heads, then we call the group that is associated with the component parts a bounding cycle for the table. However, if we have not learned yet, in our minds, to combine the parts together to form the table, then the component parts comprise a cycle that does not yet bound anything.

Holes

A **hole** is a nonbounding cycle. A **hole** is a quotient group that has yet to be formed. For example, as above, if we have been exposed to all the requisite parts of some bit of knowledge, but have yet to formulate the proper abstraction, then we have a hole in our knowledge. If we can see the trees, but not the forest, then again we have a hole in our knowledge. People often get stuck at certain mental or emotional levels because of holes in their development. A full grown adult will often be no further along than a small child in certain areas of his life experience. If it's an emotional problem, then it may be the result of a trauma that occurred at a young age that has kept them from progressing further. More commonly, however, one sees a complete arrest of mathematical development as the result of the person never having learned how to find something such as a simple common denominator of two fractions. In either case, though, the required quotient groups have yet to be constructed, and they won't be able to progress any further until the holes have been filled in. Nonetheless, once an *n dimensional perceptual group* has been formed, all the possible n+1 *dimensional perceptual groups* exist in potential and are merely waiting to be constructed by the individual. This, perhaps, sheds some light on the age old question of whether new math is created or discovered.

Nonorientability

Two *n* dimensional simplices have **incompatible orientations** if they have kernels that represent equivalence classes whose intersection is neither the null set nor the entire class. For example, if “money” & “good” define the kernel of one simplex, and “money” & “bad” define the kernel of another simplex such that the intersection of the two kernels contains only “money,” then a difficulty is created when the two simplices are part of the same perceptual group. We cannot see “money” as simultaneously good & bad in this case, as simultaneously belonging to two different equivalence classes within the same perceptual group. We cannot see an arrow simultaneously pointing up and down. To do so leads to a contradiction.

Much of our time is spent trying to reconcile contradictory simplices. This is one of the major problems of humanity. Different religions present contradictory views of the world. People subscribe to contradictory political philosophies. People have different ideas concerning morality and different ideas concerning etiquette. Everywhere we look we are challenged to reconcile incompatible orientations.

The ways in which we can resolve contradictions are many, but the following three seem to be quite common:

1. We can suppress the offending simplex. This method we call **censorship**. We can censor a book by having it banned. We can censor, through banishment or through violence, a person or group of people who have ideas contradictory to

ours. Censorship can include *war* to eliminate those viewpoints that conflict with the one preferred, *denial* of the actual existence of the contradictory views, or *separation* from the source of the conflict. Either way, the goal is to not be presented with the simplex that makes our perceptions nonorientable.

2. We can alter the boundary of one or more simplices. This method we call **redefinition**. For example, in order to reconcile different religions we can focus only on those ideas that they all have in common, and disregard the rest. This actually results in a change in the way we are defining each religion. Beliefs which lead to contradictions are removed from consciousness so that no disorientation occurs. If only one side redefines its boundaries, then we may call it *assimilation*.
3. We can create a new kernel which is large enough to contain the kernels of all the various simplices that we want in our perceptual group. This method is a particular type of redefinition that we call **transcendence**. The only way to see “money” as both “good” & “bad” simultaneously is to create an equivalence relation in which “money,” “good,” and “bad” are equivalent. If you want to see the arrow pointing both up & down, the “arrow,” “up,” and “down” must be made equivalent. In this method, good & bad, up & down, yin & yang become just two sides of the same coin. A prerequisite for transcendence, however, is to first observe all the opposing views separately for unless you have experienced all sides of the coin, you won’t be able to combine them into a single equivalence class.

As an example, suppose that one person sees religion as good and another person sees religion as bad. This means that the first person includes “religion” and “good” in the same equivalence class, and the second person, likewise, includes “religion” and “bad” in an equivalence class. In this case, the two equivalence classes contradict one another, and the two may deal with the contradiction in a variety of ways. If person 1 and person 2 mutually decide to simply never talk about religion, or if person 1 is able to impose their will on person 2 to forbid them from talking about religion, then this would be a resolution of the conflict through **censorship.** On the other hand, if either person 1 or person 2 or both change their equivalence class regarding religion so that it matches the other, then this would represent resolution through **redefinition**. However, if both person 1 and person 2 create new equivalence classes that permit religion to be both good and bad, then we would say that the conflict has been resolved through **transcendence**.

In *Pirkei Avot* (“Chapters of the Fathers”) we read the following:

> *“What is the kind of controversy that is in the name of Heaven? Such was the controversy between Hillel and Shammai. And what kind of controversy is not in the name of Heaven? Such was the controversy between Korach and all his congregation. (Pirkei Avot 5:17)”*

The two ancient rabbis Hillel and Shammai were known for their disagreements, but they argued and debated in order to arrive at a common understanding of the truth, not in order to destroy, and thus, they provide an example of how we should deal, if possible, with the contradictions in our lives. The goal is not war or censorship or even assimilation. Instead, the goal should be growth and transcendence. Granted that it takes two to tango (or tangle!), but sometimes we just have to separate ourselves from illogical arguments and move on. Keep in mind that growth is the ultimate goal, and we should try to avoid wasting our time and emotions on arguments that are pointless.

Extensions

Our perceptual groups continually change by two particular methods. One is by forming quotient groups, and the other is by forming extensions. If a new object comes into our field of awareness, then we can see it only if it corresponds to a simplex that we have previously learned to construct. There is a legend that when the Spanish first landed at Tierra del Fuego, only the shaman was capable of seeing the ships in the bay. Similarly, to the bulk of humanity, a book on abstract algebra is just a meaningless arrangement of symbols. These are examples of simplices that cannot be seen by all. However, given that an object is visible, then an extension of a given perceptual group by that object will be problem free only if its orientation is compatible with the orientations of the other simplices in the perceptual group. If the orientations are not compatible, then generally, either censorship, redefinition, or transcendence will at some point take place. Experience, though, shows that this is fairly commonplace. We often hold views that are contradictory or have to deal with others whose beliefs are incompatible with ours. For example, if two people come together who have similar points of view, then there is an immediate compatibility, and an extension can be formed that will enlarge the world of both individuals. The original *n*-dimensional perceptual groups of both individuals are extended to larger groups by the introduction of new *n*-dimensional perceptual objects. However, if two people come together with incompatible orientations and if they stay together, then first censorship will occur. They will be unable to see each other's position. Next, redefinition will occur. They will seek common ground. And eventually, transcendence might occur. They might truly seek to understand each other's position, and then extend to a higher dimensional quotient group through transcendence.

A Nonabelian Group Model

Once objects of different dimensions have been defined within the mind, we can create a nonabelian group model as follows. Suppose that the mind has defined *n* objects. Then there are *n*-factorial different permutations we may make of each object, and multiplication of these permutations by following one by another, in the usual way, forms a nonabelian group when $n \geq 3$. However, let's suppose that we also assign to each object in our *n*-tuple a number between 0 and 1, written in percent form, to represent the amount of attention we place on that object, and that we always place the object with the highest attention first and then proceed from left to right in descending order. And finally, for the sake of simplicity, let's assume that no two objects have the exact same amount of attention. In this setup, we could actually represent each element in our *n*-tuple by an ordered pair where the first element is the object and the second element is the amount of attention given to it. For example, suppose there are only three objects in our world, dog, cat, and chair, and we express those objects and their corresponding attentions as follows:

$$\left((\text{dog},95\%),(\text{chair},85\%),(\text{cat},15\%) \right)$$

This would mean that we have 95% of a theoretically possible 100% attention focused on the dog and then much more attention focused on the chair than the cat. If we equate attention with "volume" level, then you can see that it is not necessary for these percentages to add up to 100%. We can easily have the "attention volume" turned up high on all three objects. Additionally, the object with the most attention always appears first on the left, and then attention decreases as we go from left to right. Also, if we are in the process of going to sleep, then the amount of attention given to all of the objects might be well below 100%. For example, $\left((\text{dog},10\%),(\text{chair},8\%),(\text{cat},1\%) \right)$.

We can now introduce a transformation rule that seems to correspond to what happens in practice. If we start with a situation like $\left((\text{dog},95\%),(\text{chair},85\%),(\text{cat},15\%) \right)$, then we'll assume that the next perception we choose is not only chosen probabilistically, but also in a way that takes the "attention weights" into account. In other words, given our starting situation, our next perception is most likely to be the same with the focus on "dog" followed by "chair" and then "cat." However, even though there is a greater chance that "chair" will come in second again, at some point simply due to chance, we may have "chair" come in first followed by "dog" and then "cat." Similarly, due to chance fluctuations, the amount of attention given to "dog" in following perceptions could easily drop a bit, and if it keeps dropping, then at some point it could be the object receiving the least of our attention. This gives us, I believe, a group theoretic model that corresponds quite well to what happens in reality.

Preferred Reality

The fact that we all tend to organize perceptions in a similar manner suggests that there is a **preferred reality**, and hence, a **preferred orientation**. It is preferred to organize bark, leaves, and limbs into a tree, and it is preferred to organize trees into forests. The implication is that there do exist absolute rights and absolute wrongs, but one should not be hasty in using this implication as an excuse to condemn others. It does seem absolutely right to see a tree as a tree and a house as a house, but as we proceed from lower to higher dimensionional perceptual groups, reality may become more arbitrary and more a matter of personal preference. <u>The notion of a preferred reality should never be used as a justification for any kind of prejudice</u>. Nevertheless, the preferred reality, whatever it is, does seem to provide in principle a yardstick against which one's perceptions can be measured for compatibility. For example, one could jump out of a plane and see one's self as falling up, but thinking doesn't make it so. Eventually the ground will present one with an incompatible orientation. In this case, one's perception of reality has been greatly incompatible with a preferred orientation.

Preferred reality pulls on us from both ends. It conditions the way in which we initially construct our world, and it presents us with contradictions when we stray into orientations that are incompatible with it. The existence of certain common goals and ethics that may be found in the variety of cultures around the world supports the claim that we are hardwired with propensities toward a certain preferred reality. Nonetheless, we must necessarily leave it to each individual to decide for himself or herself what is compatible with the preferred reality. As for ourselves, we often use the choice of life over death, of love over hate as a guide for determining the type of reality we should construct.

Twisted Reality

Thinking and feeling may be two perspectives, two orientations, that are incompatible with each other, but neither is incompatible with the preferred reality as we understand it. One may experience both orientations and then transcend them. However, other choices may definitely be in violation of preferred reality (whatever that is). To choose bad and call it good leads to disorientation. To choose evil and call it right creates a nonorientable existence. To cause pain to children and call it joy goes against the preferred reality. To create a Holocaust and call it just goes against the preferred reality. Those who take such a path become nonorientable. Like a möbius band that has been twisted so that up cannot be distinguished from down, so are their lives twisted with respect to the preferred reality. We might call such people **the twisted ones**. We all occasionally take the wrong road and create a twist in our lives, but most of us have just minor twists that don't do the same amount of harm as a major twist. Nevertheless, whether major or minor, part of our duty in this lifetime is to engage in a process of restoration of ourselves and our world by correcting these disorientations[4].

[4] This argument is not meant to justify discrimination against anyone based upon race, religion, sexual orientation, or many other personal choices. Instead, our reference above is to activities that appear to be universally agreed upon as wrong such as flagrant deceptions in dealings with others, deliberately hurting people, and cheating others out of the fruits of their labor. To do things one knows are wrong while trying to convince oneself that they are right leads to disorientation and a twisted existence.

"Woe to those who call evil good, and good evil; who put darkness for light, and light for darkness; who put bitter for sweet, and sweet for bitter!"

(Isaiah 5:20)

The Totality of All Things

What is the totality of all things? What is the class of all sets? Some have suggested that a set is a possible perception and that every perception is a set. This, however, would exclude more than the class of all sets from being a set. It is doubtful that many items that we consider to be infinite sets under Zermelo-Frankel could ever be perceptions in the sense of being completed infinities.

Let us consider the number 1 and the set of real numbers. The number 1 is essentially an abstraction from our experience with singleton sets. In the terminology of our previous discussions, 1 is a simplex whose faces consist of all the singleton sets we have experienced. No one has experienced all singleton sets, but our experience of one element sets is vast enough that we all have a virtually identical understanding of what is meant by the number 1. Not so with more advanced concepts. For instance, consider the understanding of how to solve problems that reduce to a linear equation. Many people have no understanding at all of how to solve such equations while others may understand how to solve

$$4x + 8 = 2x + 10,$$

but not

$$(1/3)x + 3/5 = (4/9)\, x - 2/7$$

or

$$(x + 2)(x - 3) = (x + 5)(x + 1)$$

or

$$ax + b = cx + d.$$

When it comes to solving linear equations, we find more variation in people's understandings than we do in the concept of the number 1. Our understanding is based upon our experience, and that understanding changes as our experience changes.

Now let us consider the set of the real numbers. No one has seen all real numbers. Rather, we have seen several examples of the real numbers, and we have the visual image of the number line plus some sense of how to determine whether a given object is a real number. It is elements of this sort that are brought together to form that particular simplex of experience that we call the real numbers. Again, vast differences will be found among individuals in their understanding of the concept of the real numbers. Some individuals will include axioms and notions of uncountability as part of their definition while others will have an understanding of the real numbers that consists of little beyond the integers and absolutely no awareness of irrational numbers. The differences in the levels of understanding can be great.

Applying these same lines of reasoning to the class of all sets, i.e. the totality of all things, we can say that no one has seen all things, but we have each seen several things and since we have a notion of things coming together to form collections, we can abstract from what we have experienced to form the concept of everything as the sum total of all that we have seen and all that we might see. Notice that just as we can formulate a concept of the real numbers without having seen all the real numbers, so we can formulate a concept of the totality of all things without having seen all things. We also note that it is likely impossible even in principle to have seen all things since given any nontrivial n dimensional perceptual group, we always seem to be able to form, through quotient groups and extensions, nontrivial n+1 dimensional simplices, and then from these n+1 dimensional simplices and extensions, nontrivial n+2 dimensional simplices and extensions at that level, and so on and so on (an infinity that we cannot complete --- somewhat like starting with $\varnothing$ and then generating a never ending sequence of successive power sets). Also, we cannot even in principle see all possible n-dimensional simplices at once since many perceptions are mutually exclusive. If we see a blue sky above us, then we don't see a red sky above us, or a green sky, or a sky of some other color. Thus, our understanding of the totality of all things is a type of abstraction based upon the notion that things may be grouped together to form a completed collection.

For most people the idea of the totality of all things is little more than a convenient label. However, if we try to peek at the reality behind the label, then we encounter an unspeakable mystery for this reality is not one that we can perceive. Unlike being able to talk about a blue sky and then go out and look at one, the totality of all things cannot be perceived in the same way. We have the label, but not the sense perception to go along with it. To try and conjure up a perception for the totality of all things is to hit upon a brick wall, a void.

The above explains one way in which the totality of all things can be deduced to be a void (from the point of view of our inability to have a perceptual experience of the totality of all things). Now let us look at another way to arrive at this conclusion via the concept of union. Certainly the union of all things would be another way of describing the totality of all things, so let us examine further the very concept of union. On the one hand, the idea of union has the usual meaning that it takes on in set theory. In other words, take two or more collections and combine them together to form another collection, $\{a, b\} \cup \{c, d\} = \{a, b, c, d\}$. To the world at large, however, the concept of union means more than just combining two collections. The concept of union means "oneness" --- "the two become one." <u>This latter concept of union really embraces the notion of a quotient structure since for the two to become one, they must be placed within the same equivalence class</u>. Notice that set theoretic union may also be looked upon in this manner if we consider that when we take the union of two collections, we are factoring out that which keeps the two collections separate. Thus, the general concept of union is one of forming a quotient structure.

Now let us consider the totality of all things, or more realistically the totality of all things a person has experienced, as the union of all things. What would such a union mean? It would mean that all things have been united into a single equivalence class. What would such a perception look like? It would be a perception in which no distinction is made between anything because the difference between all things, including the observer and the observed, has been factored out. But this would be a state of indescribable voidness since there is nothing separate from the observer to be seen. Is there any other term that we might apply to such a state of affairs? Obviously yes since this is also what we would have to mean by the null set, for what else could the null set be from a perceptual standpoint other than a perception of no-thing[5]?

When we speak of nullity it is often a relative nullity such as an empty box or an empty room. There is emptiness with respect to the box or with respect to the room, but not an absolute emptiness since perceptions such as "box" and "room" are still present. In an absolute nullity there would be no objects present and no separation between observer and observed, and quite paradoxically this is also how the totality of all things as the union of all things would be perceived.

In many cultures and religions there has been the notion of the void that contains or gives birth to everything. In the *Kabbalah* of classical Jewish mysticism, the totality of all things is *ein* which literally means no-thing and which is beyond perception. In the *Sefer Yetzirah*, "The Book of Creation," the oldest extant work on *Kabbalah,* one finds descriptions of how the world is given birth and unfolds from unfathomable spirit. Here is an example from chapter 2 of that text.

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[5] In the so-called "perception" of the null set, if indeed we may call it a perception, all things including the observer and observed are combined. If, however, there were a perception in which an observer sees a "sea of nothingness" that contains no distinct objects, this would be different. There would still be separation between the observer and this nothingness. This would be like "the set of the null set," the set that contains the null set as its only element.

"2:6 IT formed reality from formless amazement,

and made ITS nonexistence existence,

and IT shaped great pillars from air that cannot be caught,

and this is a sign, aleph with all of them

and all of them with aleph.

IT observes and transforms,

and makes all that is formed and all spoken things One Name,

and a sign for this thing, twenty-two desires in a single body."

For an example from Taoism, consider the following passage from the *Tao te Ching*, Chapter 25:

"There is something formless and perfect,

Existing before the birth of Heaven and Earth.

How still it is! How quiet!

Abiding alone and unchanging.

It pervades everywhere without fail.

Well may it be called mother of the world.

I do not know its name,

But label it Tao."

For an example from Greek mythology, consider the following from Ovid's *Metamorphoses*:

> *"Before the ocean was, or earth, or heaven,*
>
> *Nature was all alike, a shapelessness,*
>
> *Chaos, so-called, all rude and lumpy matter,*
>
> *Nothing but bulk, inert, in whose confusion*
>
> *Discordant atoms warred, . . . Till God, or kindlier Nature,*
>
> *Settled all argument, and separated*
>
> *Heaven from earth, water from land, our air*
>
> *From the high stratosphere, a liberation*
>
> *So things evolved, and out of blind confusion*
>
> *Found each its place, bound in eternal order."*

In each passage above we find the concept of something which is unknowable and yet which contains all things. Above, we have tried to explain why it is unknowable. We simply cannot see all things in the usual way. Even the attempt to do so draws a blank and leads us back to nullity. It is only when all observations are combined into a single equivalence class and this class, in turn, combined with the observer that we can touch, up to isomorphism, upon this totality. In the logic of our spoken language, we might make a distinction between the null set and the universal class, but from the standpoint of our minds as perceiving organs, they are indistinguishable. Neither can be perceived in the usual sense. We cannot perceive absolute nullity as an external object, and we cannot perceive absolute totality as an external object. They are both nonperception. The experience of nullity looms in the background as the kernel of our perceptual group, and the occasional experience of totality as a unified whole that has been combined even with the observer gives us a taste of complete wholeness that is indistinguishable from that of nullity. Hence, from the standpoint of our minds, $\varnothing = U$.

As an example, let's consider once gain the group $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$ with elements $\left\{ \begin{matrix} (0,0,0) \\ (tree1,0,0) \\ (0,tree2,0) \\ (0,0,tree3) \\ (tree1,tree2,0) \\ (0,tree2,tree3) \\ (tree1,0,tree3) \\ (tree1,tree2,tree3) \end{matrix} \right\}$. In this group, the identity $(0,0,0)$ plays the role of both the observer and the null set, that which is not perceived as an object. Normally when we want to create an object or simplex, we just factor out a two element group consisting of the identity and the set of elements we want to combine. For example, we previously factored out $\left\{ \begin{matrix} (0,0,0) \\ (tree1,tree2,tree3) \end{matrix} \right\}$, and we argued that this resulted in a perception of the "forest" instead of the individual trees. However, if we factor out all of the elements in our original group, then the resulting factor group is isomorphic to the identity and we have just a single equivalence class that contains everything with no distinction being made between anything. $\varnothing \cong U$.

Absolute Knowledge

As we pointed out earlier, knowledge is a quotient group. Thus, the totality of all things considered as a single equivalence class that is all inclusive is also representative of all knowledge. Hence, the totality of all things is absolute knowledge.

Absolute Love

Love is union. The two become one. From this we may conclude that if the totality of all things is the union of all things, then the totality of all things also represents the most supreme love. Thus, the totality of all things is absolute love.

Knowing the Unknowable

The totality of all things is the void. The totality of all things is the unknowable. However, if there were not some way to "know" the unknowable, then all our talk of it would be just so much dust in the wind. So let us consider what we mean by knowing the unknowable and what others have done historically. First, when an object is known in the usual sense, people usually imagine a separation between the observer and the observed. We see a particular object and we sense it as something separate from that which is doing the observing. From our previous discussions, however, it is clear that the unknowable, the totality of all things, cannot be known in this manner. The observed is not the whole as long as the observer is separate from it. However, the totality of all things can be known in another way by forming the union of the known with the knower. In other words, by creating a state of no distinction in which the known is one with the knower. The immediate question that arises is can this be done? Can one achieve a state of absolutely no distinction, and if so, what would it be like? That such a state can be achieved is validated by the testimonies of those who have achieved it. That such a state is also rare is validated by the paucity of such testimonies. Nonetheless, testimonies exist from talented mystics, psychonauts experimenting with mind-altering substances or meditative techniques, and the occasional accidental enlightenment.

The totality of all things cannot be seen. The totality of all things is nonperception, and from the point of view of the mind, all nonperception is the same. This is the key that unlocks several doors. Returning to our discussions on perceptual groups, note that <u>the observer appears as the identity element within each group</u>. This association is reasonable because the identity element corresponds to what is not observed, and the observer cannot be observed as an object separate from itself. We can’t make the observer an object that is separate from that which is doing the observing. The observer is only known through our unity with it. Thus, the observer is not qualitatively different from the ultimate state, the totality of all things, and hence, by focusing on the observer, we can “perceive” nonperception. For example, I move through my world and observe myriad things, but I also periodically follow each object until it merges/disappears in consciousness. In this way each observation becomes a vehicle that carries me back to nonperception. All perceptions merge into the oneness of the observer at the moment of observation.

Another method for achieving unity is to focus on the separation between the observer and the observed. The goal is to realize that there is no separation. We only know something at the moment that it disappears into consciousness. Consequently, when we are not looking at something, it does not exist for us, and when we look at it, it is already one with awareness. The paradigm of the observed existing apart from the observer and somehow moving over time from outside to inside conscious awareness is an illusory model created by the mind. At the moment of awareness the observed is already one with consciousness. The known is always united with the knower. *Nirvana* is *samsara*, and *samsara* is *Nirvana*.

For the most part, I experience unity simply by reminding myself to experience unity. I can look at something and remind myself that it is one with me just as much as it is apart from me, and I can also lose this unity by forgetting that unity exists. The remembrance of this unity results in a transition from Buber's *I-It* relationship to an *I-Thou* relationship. Another method of finding unity in diversity is as follows. Suppose your entire perceptual group is generated by just three objects A, B, & C. Now factor out the sum A + B + C (Recall that this sum can be interpreted as, "I see A and I see B and I see C."). In this particular factor group, the individual objects A, B, and C are present in separate equivalence classes, but the totality A + B + C has been factored out. Thus, while one sees individual objects as separate from one another, the overall perception is one of unity. Additionally, in this quotient group there is a unity between an object and its complement, between *A* and *not-A*. More specifically, our equivalence classes in the quotient structure are $\begin{Bmatrix} e \\ A+B+C \end{Bmatrix}$, $\begin{Bmatrix} A \\ B+C \end{Bmatrix}$, $\begin{Bmatrix} B \\ A+C \end{Bmatrix}$, and $\begin{Bmatrix} C \\ A+B \end{Bmatrix}$. Every element is seen as one with its complement, *I* is combined with *Thou*, and the totality A + B + C is indistinguishable from the identity, consciousness, or "no perception." A nice application of quotient groups if I do say so myself! Throughout one's day, objects continually arise out of consciousness, exist in consciousness, and return to consciousness, and in this quotient structure there is a unity that permeates all differentiation. In the language of category theory, the universe is a universal object!

To speak of knowing the unknowable and perceiving nonperception obviously seems contradictory. However, the problem is more one of language rather than experience. Sometimes new words have to be created in order to be able to grok what is.

The Second Law of Thermodynamics

Energy flows from a hot place to a cold place; entropy increases. This is the inexorable second law of thermodynamics --- one of the great arrows of time. The question, though, has always arisen as to how could living, stable bundles of energy have come into existence if the second law of thermodynamics is always urging us toward lukewarmness? The answer is that the formation of complex wholes is just another way of adding to the overall entropy in the universe. For two hydrogen atoms to come together to form a helium atom, energy has to be released in the process, and this adds to the total entropy in the universe. Continuing along these lines, the second law of thermodynamics may also be the driving force behind our mental development. We have perception after perception, and eventually a considerable amount of mental energy gets tied up in the maintenance of a particular *n*-dimensional perceptual group. The second law of thermodynamics won't allow such an amount of energy to remain concentrated in one area of space forever. Eventually, something's got to give. If energy is bound up in a problem, then entropy can be increased by either letting go of the problem and returning to the life of a couch potato, or by achieving a breakthrough. The breakthrough is a quotient group. Several elements come together to form a kernel, and a new perceptual group is formed which takes less energy to maintain than the previous one as a result of combining disparate elements together into equivalence classes. Thus, we have two ways of increasing entropy. One is through disintegration and the other is through integration. If we choose disintegration, then we let our energy slowly dissipate, and our mental acuteness likewise decreases. However, if we choose integration, then we create a new quotient group by selling entropy to the universe in exchange for knowledge. It is through this latter method that the second law of thermodynamics becomes the driving force behind mental evolution in the universe.

There are other lessons also to be learned from the second law of thermodynamics. For an example, consider the life of a city. A city is maintained by there being a continual flow of energy in and out of the city. If the flow is stopped at either end, then entropy begins to increase through decay and the city dies. The same thing happens to us as individuals. Our health is maintained by there being a constant flow of energy into and out of us. If we do not continually release energy, then there is no room for new energy to come in, and we begin to decay until we die. Thus, in order to continue to live and grow, we need to give as well as receive. If we try to receive without giving, then we will no longer voluntarily increase entropy through growth, and we will have to increase it solely through decay. Energy must flow. All things must become food for something else, and so it goes until the circle is complete. The professor's knowledge is food for the student, and the student's enlightenment is food for the professor[6]. One way or another, however, entropy must increase. Our choice is merely are we going to do it in a way that leads to decay or to the formation of higher dimensional perceptual groups.

Chaos Theory

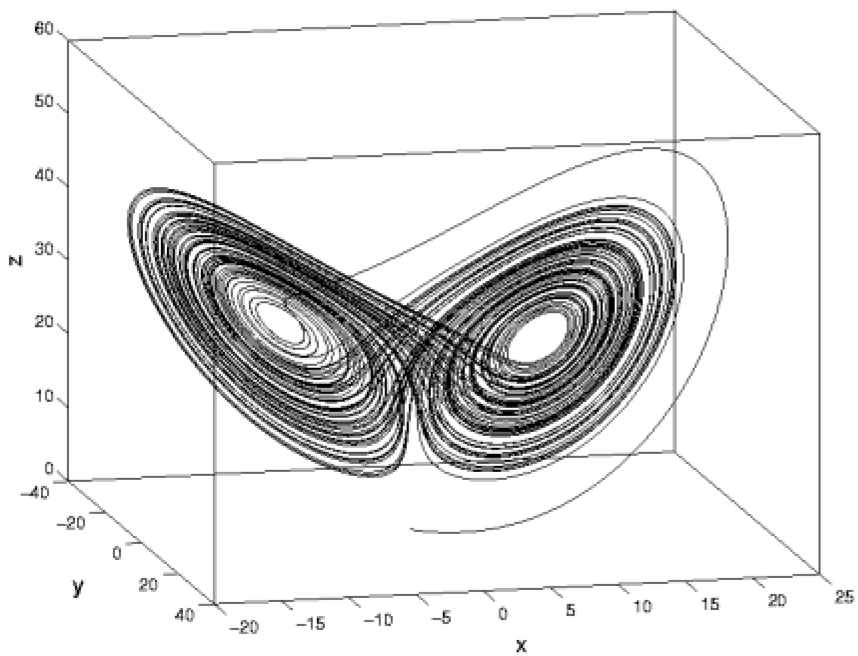
Mathematicians and scientists now understand that very complex objects can be generated by very simple rules once feedback is applied. In practical experience, this is what often happens when a simple sound is fed back through a microphone and amplifier. The output is slightly distorted from the original by each iteration until the result is very different from what we started with. In mathematics, Benoit Mandelbrot showed how we can reproduce the same situation by setting $z = 0$, fixing a number c, and continually replacing z by $z^2 + c$. Through this mathematical feedback process, complexity can ensue and mathematical fractals can be created. For example, if we set $z = 0$ and $c = -0.87$, then this process generates the following table of numbers.

[6] "Rabbi Hanina said, 'I have learned much from my teachers, more from my colleagues, but from my students I have learned the most of all.'" (Babylonian Talmud, Ta'anith 7a)

$\underline{z = z^2 + c}$

0
-0.870000
-0.113100
-0.857208
-0.135194
-0.851723
-0.144569
-0.849100
-0.149029
-0.847790
-0.151252

In this table we see numbers that get close to repeating themselves, but which fail to repeat themselves exactly. An imperfect cycle is generated, and this is often what happens with thought. In quantum mechanical terms, objects of perception make that transition from "wave" to "particle" form as a result of our intent to focus upon a particular experience. That perception then immediately becomes part of a feedback loop. The object plus our understanding of it are the output that immediately becomes input for the next iteration. Consequently, as we focus on an object or thought, it evolves and mutates before our very eyes. <u>What we focus on in our lives is what will evolve, change, and grow within our lives.</u> We turn it over and over in our minds, and variations continue to be generated until intent leads us to focus on something else. The result is analogous to a *strange attractor* such as the well-known *Lorentz attractor* where a general pattern emerges even though no part of the process is ever repeated exactly.



Often times we look back at our lives and it seems that even though at the time everything happened randomly, there were really grand patterns which guided the development of our lives. These patterns can perhaps best be understood in terms of attractors and repellors. For example, an early affinity for mathematics can result in an attractor which can eventually lead one into graduate school and a life of research. And in retrospect, it often seems like the beginning was created just so that one could arrive at a particular end. Likewise, there are repellors in our lives that correspond to areas of experience that we avoid. For example, in my case, Brussels sprouts. There are also items that for us may flip-flop back and forth between being attractive and repelling. Such items create in us what psychologists have referred to as approach/avoidance conflicts. Such love/hate relationships often occur between siblings. A person may attract us one minute, and then repel us the next. Such are the practical jokes of the universe.

Attractors form the hidden agendas in our lives ---- areas to which we constantly return. The repellors in our lives, on the other hand, represent areas that we avoid. But this also means that we will have holes in those particular areas of our lives. Eventually, we may have to go back and fill in those holes so that we can continue to advance. Also, since the second law of thermodynamics drives us toward the formation of higher dimensional perceptual groups, and since the totality of all things is the ultimate quotient group in which all things are made equivalent, we could say that union with this whole is the ultimate attractor for the universe. In such a direction are we moving, and through such thoughts can we make some sense of the chaotic Scylla and Charybdis that we swim between.

Affine transformations

Our perceptual groups do not remain constant. They are in a continual state of flux. However, for obvious reasons, some degree of stability is desired, and this suggests rules for transformation that help to preserve structure. In group theory, the appropriate structure-preserving transformations are *homomorphisms* while in linear algebra the analogous structure preserving change is known as a *linear transformation*. Furthermore, if we combine a *linear transformation* with the addition of a constant vector, then we arrive at what is known as an *affine transformation*. In the 1980s, mathematician Michael Barnsley illustrated how *affine transformations* may be used to create a variety of familiar looking fractal images resembling images found in the real world. These resulting images are the attractors associated with the corresponding transformations. In our group theory model, however, what we want to consider are *homomorphisms* and groups that are extended by adding our new objects onto the old group. In this context, we could consider all the possible *homomorphisms* and *quotient structures* and *objects* created, *homomorphisms* plus *objects* added, as defining an attractor that is associated with our process. In particular, if we focus on just a few group elements and hold the others constant in our transformations (such as focusing, for example, only on areas of mathematics), then the objects and structures that result form a type of *orbit* for this activity, and we can think of this set of all possible results as our attractor. Likewise, we can think of objects and structures that won't be reached by our processes as corresponding repellors. For example, focusing entirely on mathematics will lead me toward certain results and away from others. And as in John Conway's mathematical game of *Life*, simple rules for transformation can lead to great richness and complexity.


Final Remarks

In this paper, we have presented what is in many respects a fairly simple model of cognition, and yet it explains several things. In particular, from the model we are able to glean the following:

- Wisdom/Understanding/Knowledge is the result of the formation of quotient structures.
- Precise meaning can be given, via quotient structures, to higher dimensional thinking.
- Before we can formulate a higher dimensional structure, all of the component pieces must be present.
- If the component parts are present, but we haven't formed the corresponding quotient structure, then we have a hole in our thinking.
- If we have formed a quotient structure, then there may sometimes still be difficulty in transitioning back to the lower dimensional perceptual group. For example, simply because a person has difficulty articulating a concept, that doesn't necessarily mean they don't understand the concept. Instead, they may not be in the habit of breaking down the concept into all of its component parts.
- The model also illuminates the connection between "math created" versus "math discovered."
- In order to keep growing, we not only need to achieve higher dimensional thinking through quotient groups, we also need to extend the horizons of our perceptual groups through the introduction of new elements and experiences.

- Many contradictions that we experience in our lives may be successfully reconciled through transcendence.
- Other contradictions are violations of preferred reality (whatever that is) and can lead to perceptual structures that are nonorientable.
- Union can be described in terms of quotient groups.
- A consequence of the model and our mind's intrinsic understanding of "union" is an explanation of the relationship between the observer and the totality of all things.
- From the standpoint of the mind, $\varnothing = U$.
- The relationship between oneness, absolute knowledge, and absolute love, as understood by mystical traditions around the world, is explained by the model.
- The laws of thermodynamics propel us toward higher dimensional thinking.
- We continue to grow, via the laws of thermodynamics, though a constant interchange of information and ideas with our peers.
- Simple, structure preserving rules of transformation can lead to great complexity.
- Rules of transformation, when applied to the model, can lead to attractors and repellors.
- And our attractors and repellors, in turn, can determine the destiny of our life.

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