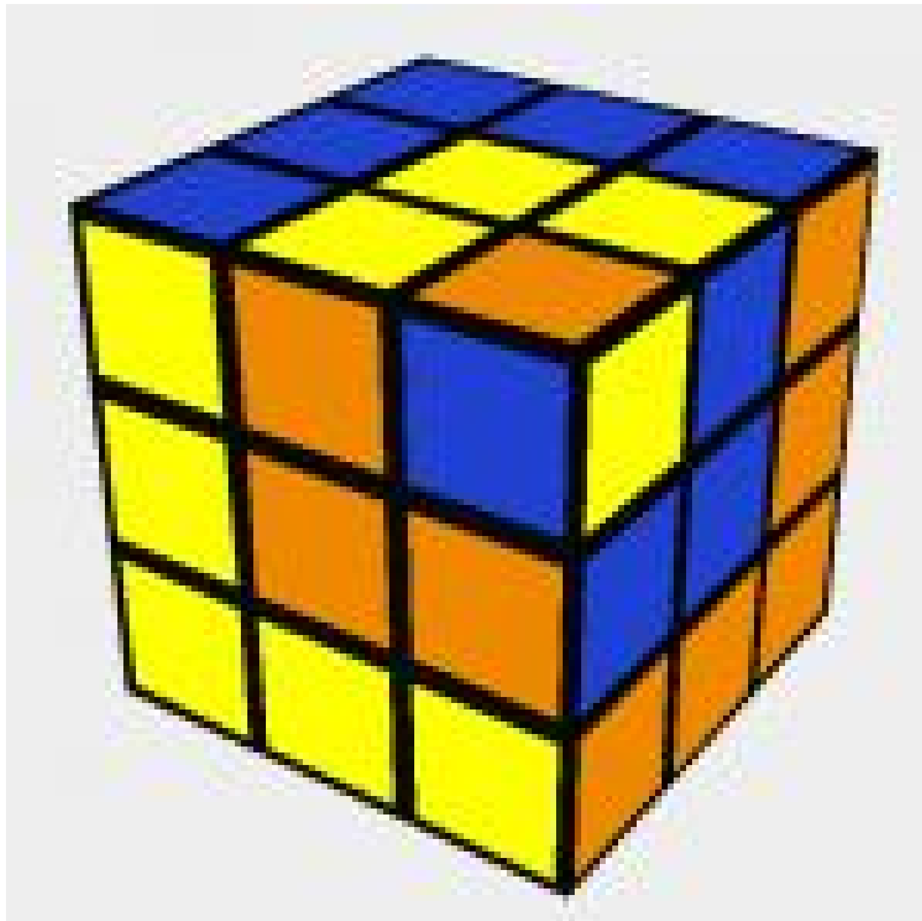
PATTERNS ON RUBIK'S CUBE



We now just want to examine a few interesting patterns you can create on the surface of Rubik's cube. However, our interest goes beyond just art. There are certainly many places where one can go and instantly find algorithms for all sorts of patterns for the cube but we want to do more than that. As usual, we want to explain some of the math that comes with these patterns.

1. This first pattern is one of my favorites. In this case, the algorithm simply switches two front edge cubes with the corresponding back edge cubes. It's simple, but elegant Also, this algorithm generates a cyclic group of order 2, and that means that the algorithm is its own inverse.

$$\left(R^2U^2\right)^3$$

2. This next pattern is created using elements of the slice group. Recall that the slice group effectively moves only the center slices of the cube, and thus, the corner cubes stay fixed. Consequently, a lot of nice patterns can be created using only slices. In the pattern below we have 6 dots centered on backgrounds of different colors. Notice that this element of the slice group has order 3.

$$UD^{-1}RL^{-1}FB^{-1}UD^{-1}$$

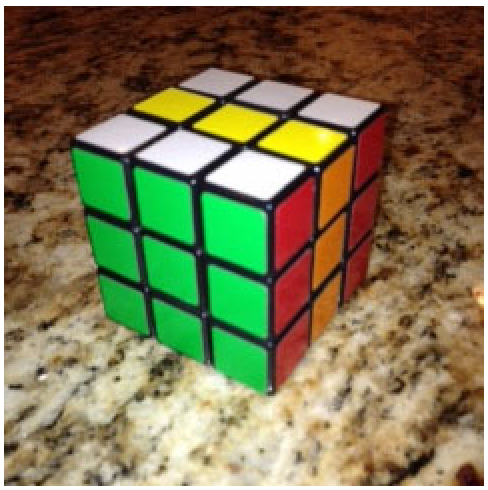
3. This next pattern of six checkerboards comes from the slice squared group. Also, recall that this group is abelian.

$$U^2 D^2 R^2 L^2 F^2 B^2$$

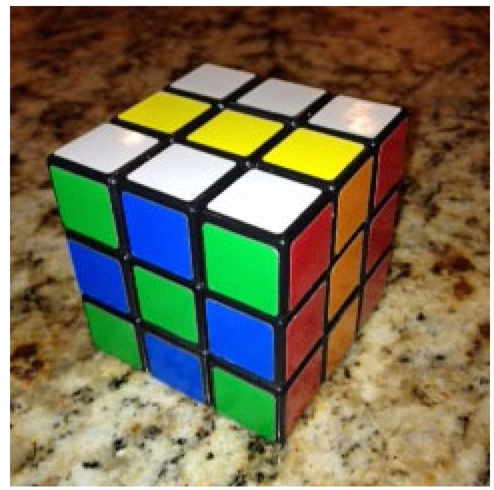
4. In this pattern, we've taken the previous pattern and created a conjugate of the form xyx^{-1}. Multiplying all elements of the group in such a manner produces a function called an *inner automorphism*, and automorphisms preserve a lot of the internal structure of the group. With luck, this will also transform one interesting pattern into another. In this case, we see our previous pattern of six checkerboards transformed into one of four checkerboards.

$$\left(R^2U^2\right)^3 U^2D^2R^2L^2F^2B^2\left(R^2U^2\right)^3$$

5. The next four pictures are going to be based on patterns from the slice squared group. We'll first look at a pattern from this group, and then we'll form a conjugate of that pattern using $\left(R^2U^2\right)^3$.



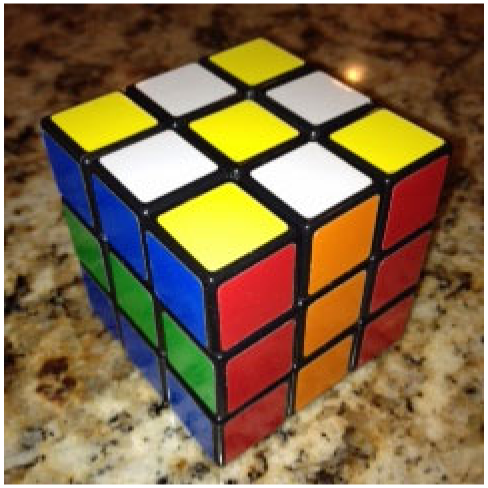
$$F^2B^2$$



$$\left(R^2U^2\right)^3 F^2B^2\left(R^2U^2\right)^3$$

6. This time we'll start with $R^2L^2F^2B^2$ and then, again, form a conjugate with $(R^2U^2)^3$. $(R^2U^2)^3 R^2L^2F^2B^2 (R^2U^2)^3$

$R^2L^2F^2B^2$



$(R^2U^2)^3 R^2L^2F^2B^2 (R^2U^2)^3$

7. This one is one of my favorites. It creates a center dot on two of the faces, a checkerboard on two, and stripes on the remaining two.

$$\left(R^2U^2\right)^3 R^2L^2F^2B^2\left(L^2U^2\right)^3$$

8. If you start with the red face in front and the white face to the right, then this will create 4 crosses with one of them a red Templar cross on a white background. I like this one because a few of my ancestors on my dad's side were Knights Templar.

$$LUFLULDLDU^{-1}F^{-1}U^{-1}F^{-1}D^{-1}F^{-1}L^{-1}D^{-1}F^{-1}$$

9. Again start with the red face in front and the white fact to the right. This algorithm will produce 6 crosses with one of them a red Templar cross on a white background.

$$L^2 R^{-1} F D^2 L^{-1} F^{-1} R L^{-1} F B^{-1} L F U^2 L^{-1} F^2 B$$

10. One thing I like to do is to see what new patterns I can create by combining

$UD^{-1}RL^{-1}FB^{-1}UD^{-1}$ with $\left(R^2U^2\right)^3$. See what interesting things you can come up with!

Start with $UD^{-1}RL^{-1}FB^{-1}UD^{-1}$, rotate the whole cube, and do $UD^{-1}RL^{-1}FB^{-1}UD^{-1}$ again to get a pattern of four dots.

Take the 4-dot pattern and add $\left(R^2U^2\right)^3$, rotate the whole cube, and do $\left(R^2U^2\right)^3$ again. You should now have a 4-dot pattern combined with a checkerboard pattern!

11. And finally, this algorithm creates an incredible pattern called the superflip. Basically, every cubelet is in its home position, but every single edge cubelet has been flipped. It's pretty easy to see that the group generated by this move has order 2, but what is not so obvious is that this element of the Rubik's cube group commutes with every other element of that group. In fact, the only other element in the group that does that is the identity. In group theory, the set of all elements of a group that commute with every other element is called the center of the group and the center of the Rubik's cube group consists of only the identity and the superflip.

$$UR^2FBRB^2RU^2LB^2RU^{-1}D^{-1}R^2FR^{-1}LB^2U^2F^2$$

or

$$FLULB^{-1}U^{-1}D^{-1}LF^{-1}U^{-1}B^{-1}RL^{-1}BF^2U^{-1}D^{-1}F^2B^2R^2U^{-1}D^{-1}$$