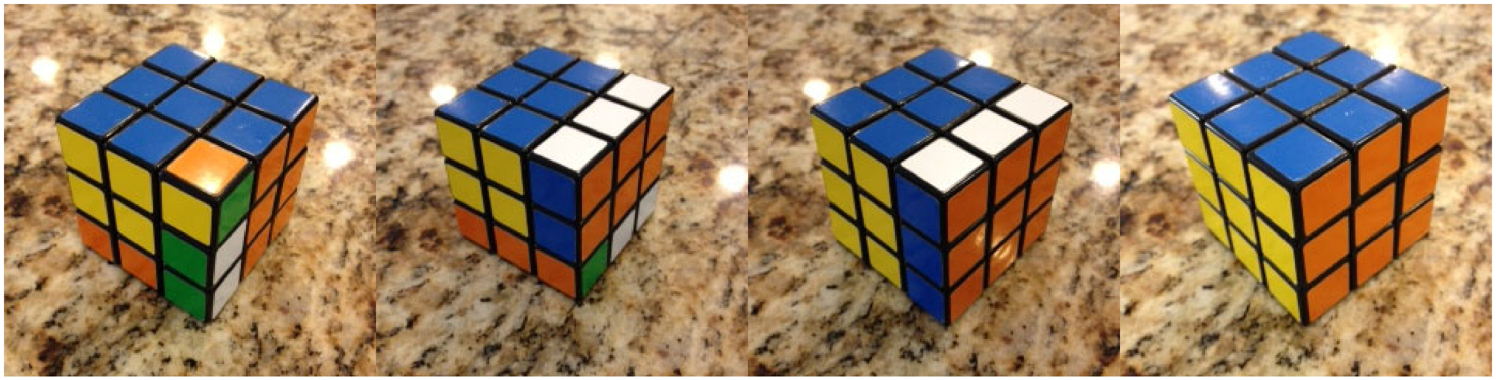
CONJUGATE AND COMMUTATORS
IN
RUBIK'S CUBE

$$R^{-1}DR$$

Many people have either written or talked about how the cube can mostly be solved using either conjugates or commutators, and so let's look at a few ways in which we are already using them. First, at the very beginning when I am trying to correctly place the corners on the top face of the cube, I often do the move $R^{-1}DR$ which is a conjugate. Thus, let's think about what this move does for us. My goal with this move is to place something in the up-front-right (UFR) corner. To do this, I position the cubelet that I want to move there in the down-front-left (DFL) corner. Then I do R^{-1}. That moves the up-front-right (UFR) corner to the down-front-right (DFR) position. Next, I do D, and this moves my cubelet from DFL to DFR. And finally, I do R, and that rotates my cublet from the down-front-right (DFR) position back into the up-front-right (UFR) corner that was my goal. In a nutshell, you can say that we shifted things from the top face to a workspace down below, moved something into the workspace, and then moved it back to the top row.



$$R^{-1}DR$$

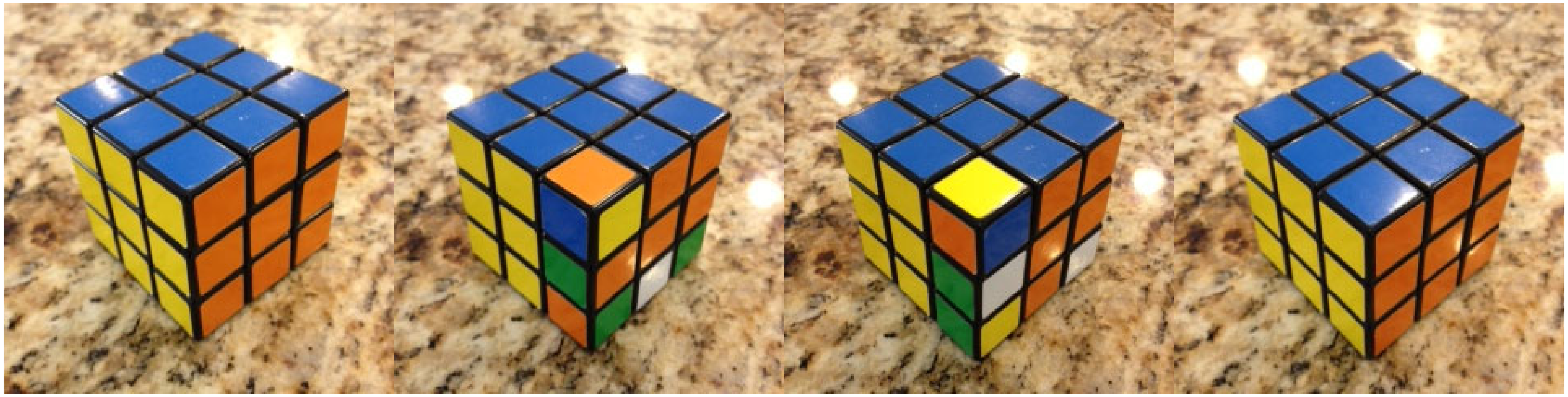
When we do a commutator on Rubik's cube, the idea is that we are, most of the time, partially undoing what we have previously done. In particular, what happens when we do a commutator like $R^{-1}D^{-1}RD$ on the cube is that some of the cubelets get move around, but others stay right where they are, and anytime we move just a few cubelets, that gives us a tool we can use for easily solving the cube.

Thus, let's examine this particular commutator, $R^{-1}D^{-1}RD$, in greater detail. It, by the way, is the move that we usually use at the end to get our final corner cubelets turned correctly. If we do this move just once, then we'll transpose two sets of corner cubelets, and we'll cycle three edge cubelets,

$$\left(DB \quad DR \quad FR\right)\left(DRF \quad UFR\right)\left(DBR \quad DLB\right)$$

Thus, if we repeat this operation a second time, then we'll restore the corner cubelets and just cycle the edge cubelets. However, when I do this, my corner cubelets, while being in the right corners, have also been twisted counterclockwise through angles that are multiples of $120°$. And as you might suspect from the presence of 3 & 2-cycles that we have above for $R^{-1}D^{-1}RD$, repeating this move 6 times, the least common multiple of 2 and 3, will finally restore everything back to its starting point.

And now you can see why this is our finishing move for the cube. At the end, we have all the cubelets in their correct positions, but some of the corner cubelets on top are usually twisted. To untwist them, we apply the algorithm $(R^{-1}D^{-1}RD)^2$ until we get one corner untwisted. Then we rotate the top to move another twisted corner into position, and we repeat with $(R^{-1}D^{-1}RD)^2$ until that one is untwisted. However, the one thing that we are mathematically guaranteed is that the number of times we have to do $(R^{-1}D^{-1}RD)^2$ is always going to be some multiple of 3. Thus, suppose we have to do $(R^{-1}D^{-1}RD)^2$ just three times. Then $[(R^{-1}D^{-1}RD)^2]^3 = (R^{-1}D^{-1}RD)^6 = e$. In other words, since our algorithm has order 6, by the time we are done untwisting the cubelets, everything has been returned to its proper position and orientation.



$$(R^{-1}D^{-1}RD)^2, (R^{-1}D^{-1}RD)^4, (R^{-1}D^{-1}RD)^6$$