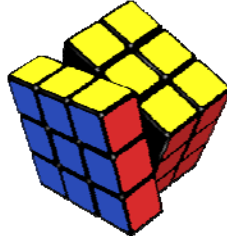


## Lesson 13

### INTRODUCTION TO THE STABILIZER SUBGROUP

If we have a group  $G$  that acts on a set of points, then the orbit of that point is the set of all points that our starting point can be moved to by the permutations in  $G$ . Complementary to this idea is that of the stabilizer of our point. In this instance, we want to find all the permutations in  $G$  that leave our point fixed. For example, if  $X = \{1, 2, 3, 4, 5, 6\}$  and if

$G = \{(), (1, 2), (3, 4), (5, 6), (1, 2)(3, 4), (1, 2)(5, 6), (3, 4)(5, 6), (1, 2)(3, 4)(5, 6)\}$ , then the stabilizer of 1 in this group is  $Stabilizer(G, 1) = \{(), (3, 4), (5, 6), (3, 4)(5, 6)\}$ . Notice that the stabilizer of 1 is going to be a subgroup of  $G$  because if we take any two permutations that stabilize 1, then their product (one permutation followed by the other) will also stabilize 1. Hence, the set of stabilizers of 1 in  $G$  is closed under multiplication. Also, if a permutation stabilizes 1, then its inverse must also stabilize 1. Hence,  $Stabilizer(G, 1)$  is a subgroup of  $G$ . We can extend this to the stabilizer of a list or  $n$ -tuple of points. For example, the set of all permutations in  $G$  that stabilize both 1 and 3 is something that we'll write as  $Stabilizer(G, [1, 3], OnTuples) = \{(), (5, 6)\}$ . And as before,  $Stabilizer(G, [1, 3], OnTuples)$  is a subgroup of  $G$ . By the way, the notations we are using here correspond to how you would write these commands in the free computer program *GAP* (*Groups, Algorithms, and Programming*) in order to find these stabilizers.



In something like the solution to Rubik's cube, stabilizers are important because if we have already solved the bottom layer, then we want to know what permutations we can do that leave the bottom layer untouched. In other words, we want to proceed with permutations that stabilize the bottom layer. And once we solve the bottom and middle layer, then we basically want to continue with permutations that stabilize, or leave untouched, these two layers.