

## Lesson 10

### INTRODUCTION TO CONJUGATES – ANSWERS

Below is a group  $G$  and two subgroups,  $H_1$  and  $H_2$ . For each  $b$  given below, find  $H_1^b = b^{-1}H_1b$  and  $H_2^b = b^{-1}H_2b$ .

$$G = \left\{ \begin{array}{l} () \\ (1,2,3) \\ (1,3,2) \\ (2,3,4) \\ (2,4,3) \\ (1,2,4) \\ (1,4,2) \\ (1,3,4) \\ (1,4,3) \\ (1,2)(3,4) \\ (1,3)(2,4) \\ (1,4)(2,3) \end{array} \right\} \quad \text{and} \quad H_1 = \left\{ \begin{array}{l} () \\ (1,2,3) \\ (1,3,2) \end{array} \right\} \quad \text{and} \quad H_2 = \left\{ \begin{array}{l} () \\ (1,2)(3,4) \\ (1,4)(2,3) \\ (1,3)(2,4) \end{array} \right\}$$

1.  $b = (1,3)(2,4)$

$$b^{-1}H_1b = (1,3)(2,4) \left\{ \begin{array}{l} () \\ (1,2,3) \\ (1,3,2) \end{array} \right\} (1,3)(2,4) = \left\{ \begin{array}{l} (1,3)(2,4)() (1,3)(2,4) \\ (1,3)(2,4)(1,2,3)(1,3)(2,4) \\ (1,3)(2,4)(1,3,2)(1,3)(2,4) \end{array} \right\} = \left\{ \begin{array}{l} () \\ (1,3,4) \\ (1,4,3) \end{array} \right\}$$

$$b^{-1}H_2b = (1,3)(2,4) \left\{ \begin{array}{l} () \\ (1,2)(3,4) \\ (1,4)(2,3) \\ (1,3)(2,4) \end{array} \right\} (1,3)(2,4) = \left\{ \begin{array}{l} (1,3)(2,4)() (1,3)(2,4) \\ (1,3)(2,4)(1,2)(3,4)(1,3)(2,4) \\ (1,3)(2,4)(1,4)(2,3)(1,3)(2,4) \\ (1,3)(2,4)(1,3)(2,4)(1,3)(2,4) \end{array} \right\} = \left\{ \begin{array}{l} () \\ (1,2)(3,4) \\ (1,4)(2,3) \\ (1,3)(2,4) \end{array} \right\}$$

2.  $b = (2,3,4)$

$$b^{-1}H_1b = (4,3,2) \left\{ \begin{array}{l} () \\ (1,2,3) \\ (1,3,2) \end{array} \right\} (2,3,4) = \left\{ \begin{array}{l} (4,3,2)() (2,3,4) \\ (4,3,2)(1,2,3)(2,3,4) \\ (4,3,2)(1,3,2)(2,3,4) \end{array} \right\} = \left\{ \begin{array}{l} () \\ (1,3,4) \\ (1,4,3) \end{array} \right\}$$

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$$b^{-1}H_2b = (4,3,2) \left\{ \begin{array}{l} () \\ (1,2)(3,4) \\ (1,4)(2,3) \\ (1,3)(2,4) \end{array} \right\} (2,3,4) = \left\{ \begin{array}{l} (4,3,2)() (2,3,4) \\ (4,3,2)(1,2)(3,4)(2,3,4) \\ (4,3,2)(1,4)(2,3)(2,3,4) \\ (4,3,2)(1,3)(2,4)(2,3,4) \end{array} \right\} = \left\{ \begin{array}{l} () \\ (1,3)(2,4) \\ (1,2)(3,4) \\ (1,4)(2,3) \end{array} \right\}$$