

Lesson 11

INTRODUCTION TO COMMUTATORS – PRACTICE

Let $a = (1, 2, 3)$ and let $b = (2, 3)$, and define the commutator of a by b by $[a, b] = a^{-1}b^{-1}ab$.

Notice that $a^{-1} = (3, 2, 1)$ and $b^{-1} = (3, 2) = (2, 3) = b$. Now find the following:

1. $[a, b]$
2. $[b, a]$
3. $[a^{-1}, b^{-1}]$
4. $[b^{-1}, a^{-1}]$
5. Let $C_2 \times C_2 = \{(), (1, 0), (0, 1), (1, 1)\}$ be the Klein 4-group. Find the commutator subgroup of $C_2 \times C_2$.
6. Let $S_3 = \{(), (1, 2), (1, 3), (2, 3), (1, 2, 3), (3, 2, 1)\}$ be the symmetric group of degree 3. Find the commutator subgroup of S_3 .