Lesson 11

INTRODUCTION TO COMMUTATORS – ANSWERS

Let $a=(1,2,3)$ and let $b=(1,2)$, and define the commutator of a by b by $[a,b]=a^{-1}b^{-1}ab$. Notice that $a^{-1}=(3,2,1)$ and $b^{-1}=(1,2)=(2,1)=b$. Now find the following:

1. $[a,b]$

   $[a,b]=a^{-1}b^{-1}ab=(3,2,1)(1,2)(1,2,3)(1,2)=(1,2,3)$

2. $[b,a]$

   $[b,a]=b^{-1}a^{-1}ba=(1,2)(3,2,1)(1,2)(1,2,3)=(1,3,2)$

3. $\left[a^{-1},b^{-1}\right]$

   $\left[a^{-1},b^{-1}\right]=aba^{-1}b^{-1}=(1,2,3)(1,2)(3,2,1)(1,2)=(1,3,2)$

4. $\left[b^{-1},a^{-1}\right]$

   $\left[b^{-1},a^{-1}\right]=bab^{-1}a^{-1}=(1,2)(1,2,3)(1,2)(3,2,1)=(1,2,3)$

5. Let $C_2\times C_2=\{(\ ),(1,0),(0,1),(1,1)\}$ be the Klein 4-group. Find the commutator subgroup of $C_2\times C_2$.

   Since $C_2\times C_2$ is abelian, it follows that its commutator subgroup is the identity, $e=(\ )$.

6. Let $S_3=\{(\ ),(1,2),(1,3),(2,3),(1,2,3),(3,2,1)\}$ be the symmetric group of degree 3. Find the commutator subgroup of S_3.

   By examination we find that every commutator involving two non-identity elements is a member of the subgroup $\{(\ ),(1,2,3),(3,2,1)\}$. Hence, the subgroup generated by the commutators is also $\{(\ ),(1,2,3),(3,2,1)\}$.