INDEX OF THEOREMS

1. A group G has a unique identity element. In other words, it has only one element e with the property that for every $a \in G$, $e \cdot a = a = a \cdot e$.

2. Let G be a group, and let $a,b,c \in G$. If $ab = ac$, then $b = c$.

3. Let G be a group, and let $a,b,c \in G$. If $ba = ca$, then $b = c$.

4. Let G be a group, and let $a \in G$. Then a has a unique inverse, denoted by a^{-1}.

5. Let G be a group, and let $a \in G$. Then $a = (a^{-1})^{-1}$.

6. Let G be a group, and let $a,b \in G$. Then $(ab)^{-1} = b^{-1}a^{-1}$.

7. Let G be a group. If $x^2 = e$ for every $x \in G$, then G is abelian.

8. Let G be a group and let $a, b \in G$. If $ab = e$, then $ba = e$.

9. Let G be a group and let H be a subset of G. If for every $a \in H$ we have that $a^{-1} \in H$ and if for every $a, b \in H$ we have that $ab \in H$, then H is a subgroup of G.

10. Let G be a finite group and let H be a subset of G. If for every $a, b \in H$ we have that $ab \in H$, then H is a subgroup of G.

11. If H is a subgroup of a group G, then any two right (left) cosets either coincide or have an empty intersection.

12. If H is a subgroup of a finite group G, then any two right (left) cosets have the same number of elements.

13. If H is a subgroup of a finite group G, then the order of H is a divisor of the order of G.

14. If H is a subgroup of a finite group G, then the number of right (left) cosets of H in G, denoted by $[G:H]$, is equal to $\frac{|G|}{|H|}$.

15. If H is a subgroup of a finite group G, then $HH = H$.

16. If H is a subgroup of a group G, then the right (left) cosets of H in G define an equivalence relation.

17. If H is a normal subgroup of G and $Ha_1 = Ha_2$ and $Hb_1 = Hb_2$, then $Ha_1b_1 = Ha_2b_2$.

18. If H is a subgroup that is not a normal subgroup of G and $Ha_1 = Ha_2$ and $Hb_1 = Hb_2$, then Ha_1b_1 is not necessarily equal to Ha_2b_2.

19. If N is a normal subgroup of a group G, then $G/N = \{Na \mid a \in G\}$ is a group where the multiplication of cosets is defined in terms of the multiplication of elements in G. In other words, $Na \cdot Nb = N(ab)$.

20. The center of a group G is a normal subgroup of G.

21. The commutator (or derived) subgroup of a group G is normal in G.

22. Let G be a group of permutations. Then the set of all even permutations in G form a normal subgroup.

23. If H is a subgroup of a group G, then the subgroup N generated by H and its conjugates is normal in G.

24. If a finite group G has an even number of elements, then at least one non-identity element is its own inverse.

25. Let G be a group, let M and N be normal subgroups of G, and let $m \in M$ and $n \in N$. Then the commutator of m by n, $m^{-1}n^{-1}mn$, is an element of $M \cap N$.

26. Let G be a group, let M and N be normal subgroups of G such that $M \cap N = e$ (the identity), and let $m \in M$ and $n \in N$. Then m and n commute with one another, or in other words, $mn = nm$.

27. Let G be a group, let M and N be normal subgroups of G such that $MN = G$ and $M \cap N = e$ (the identity). Then if $m_1, m_2 \in M$ and $n_1, n_2 \in N$ such that $m_1n_1 = m_2n_2$, it follows that $m_1 = m_2$ and $n_1 = n_2$. In other words, each element in G can be represented in a unique way as a product of an element in M with an element in N.

28. If M and N are normal subgroups of G such that $M \cap N = e$ and $G = MN$, then G is isomorphic to the direct product of M and N, $G \cong M \times N$.

29. If H is a subgroup of a group G and if N is a normal subgroup of G, then the right (left) cosets corresponding to elements of H form a subgroup of G/N.

30. If *H* is a normal subgroup of a group *G* and if *N* is a normal subgroup of *G*, then the right (left) cosets corresponding to elements of *H* form a normal subgroup of G/N.

31. Let *G* be a group, *N* a normal subgroup of *G*, and let *M* be a subgroup of G/N that contains *N*. Also, define $f: G \to G/N$ by $f(g) = Ng$, and define $f^{-1}: G/N \to G$ by $f^{-1}(Ng) = \{g \in G \mid f(g) \in Ng\}$. Similarly, for any set $A \subseteq G/N$ let $f^{-1}(A) = \{g \in G \mid f(g) \in A\}$. Then if *M* is a subgroup of G/N, $f^{-1}(M)$ is a subgroup of *G*.

32. Let *G* be a group, *N* a normal subgroup of *G*, and let *M* be a subgroup of G/N that contains *N*. Also, define $f: G \to G/N$ by $f(g) = Ng$, and define $f^{-1}: G/N \to G$ by $f^{-1}(Ng) = \{g \in G \mid f(g) \in Ng\}$. Similarly, for any set $A \subseteq G/N$ let $f^{-1}(A) = \{g \in G \mid f(g) \in A\}$. Then if *M* is a normal subgroup of G/N, $f^{-1}(M)$ is a normal subgroup of *G*.

33. Every finite group *G* is isomorphic to a group of permutations acting on a set of objects.

34. Let *G* be a group, let $g \in G$, and define a function $T_g: G \to G$ by $T_g(x) = gxg^{-1}$. Then $T_g: G \to G$ is a one-to-one and onto function, or in other words, a bijection.

35. Let *G* be a group, let $g \in G$, and define a function $T_g: G \to G$ by $T_g(x) = gxg^{-1}$. Then T_g is an isomorphism.

36. Every group *G* is isomorphic to a group of permutations acting on a set of objects. (2^{nd} Proof)

37. Given a group *G*, a subgroup *H*, and a set *M* equal to all the subgroups conjugate to *H*, then the subgroup generated by elements of *M* is a normal subgroup of *G*.

38. If *G* acts on a set *X* and if $x \in X$, then the stabilizer of *G* on x is a subgroup of *G*.

39. If *G* acts on a set *X*, then every permutation in *G* is either even or odd, but not both.