HOMOMORPHISMS AND INVERSES

Discussion: This theorem takes care of another housekeeping detail. It shows us that any homomorphism from one group to another always pairs an inverse element in the first group with the corresponding inverse element in the second group. Again, it's not difficult to prove, but you gotta take the time to verify it anyway.

Theorem: Let *A* be a group, let $f : A \to B$ be a homomorphism from *A* onto *B*, and let $a \in A$. Then $f(a^{-1}) = f(a)^{-1}$. In other words, the inverse of *a* in *A* gets mapped to the inverse of $f(a)$ in *B*.

Proof: Clearly, $e = f(e) = f(aa^{-1}) = f(a)f(a^{-1})$ implies that $f(a^{-1}) = f(a)^{-1}$.

□