HOMOMORPHISMS AND IDENTITIES

<u>Discussion:</u> This theorem just takes care of some housekeeping details. It shows us that any homomorphism from one group to another always pairs the identity element in the first group with the identity element in the second group. It's not difficult to prove, but you gotta take the time to verify it anyway.

<u>Theorem:</u> Let *A* be a group and let $f : A \to B$ be a homomorphism from *A* onto *B*. Then $f(e) = e$.

<u>Proof:</u> Technically, we should perhaps denote the identity element in *A* by e_A and the identity element in *B* by e_B, but it is much more convenient to use *e* as the generic symbol for any identity element, and usually little confusion arises by using *e* to represent both identities. Thus, let $x \in A$. Then $f(x) = f(e \cdot x) = f(e)f(x)$. Now just multiply both sides of this equation by $f(x)^{-1}$ to obtain $f(e) = e$.

□