Lesson 23

GROUP THEORY AS AN ACT OF CREATION

The oldest existing book of Jewish mysticism is a short text called "The Book of Creation" (*Sefer Yetzirah* in Hebrew) which is at least 1800 years old, and this work outlines two types of creative processes, something-from-something creation (*yesh m'yesh*) and something-from-nothing creation (*yesh m'ayin*). Interestingly, both of these processes relate to group theory, and so we'll give a brief description of them here.

Something-from-something creation refers to just making a permutation out of what is already present, and by now we know quite a bit about how permutations can multiply and combine to form groups. And if you examine your life, you might quickly realize that a great deal of your time is spent on creating permutations of what's in your environment. For instance, if you clean up a room or sweep the floor or put dishes in a dishwasher, then all you are doing is creating a permutation of what already exists. Or as I like say, the only difference between a clean room and a dirty room is simply how things are arranged. Additionally, if you go to a job, then regardless of whether you are making hamburgers or typing up reports, you are likely spending a lot of your time engaged in something-from-something creation, simply moving things around, and the whole point of this type of creation is generally to repair things in order to make the world a better place.

The anonymous author of "The Book of Creation" understood some of this science of counting permutations, too, when he (or she) wrote:

"Two stones build 2 houses, three stones build 6 houses, four stones build 24 houses, five stones build 120 houses, six stones build 720 houses, seven stones build 5040 houses. From here, henceforth, go and consider what the mouth is not able to speak and the ear is not able to hear."

In group theory, of course, we would state this by saying that if you have *n* objects, then the group consisting of all permutations of those *n* objects has size $n! = n(n-1)(n-2)\ldots 1$, and we call this the symmetric group of degree *n*. What I particularly like about this ancient text, though, is that they got their counting right. In other words, $2! = 2$, $3! = 6$, $4! = 24$, $5! = 120$, $6! = 720$, and $7! = 5040$. Ancient peoples weren't stupid! In fact, they were often quite brilliant.

The second type of creation, something-from-nothing creation, is a little bit more involved, and "The Book of Creation" breaks up this creating process into five basic steps that, when translated from the Hebrew, can be described as *decreeing, shaping, combining, weighing,* and *exchanging*. The first step, *decreeing*, is an act of will or intent, and it can be something as simple as telling yourself that you want to write a song or prove a theorem or start a new business. Whatever the content, this type of creation always begins with an intent to accomplish something. The second step, *shaping*, refers to that stage where we are mulling things over in our mind as we ponder how we are going to accomplish our goal.

It is the third step, *combining*, that involves group theory. Recall in our discussion of quotient groups, each of which corresponds to a normal subgroup of some larger group, we pointed out that in the equivalence classes or cosets of our quotient group, the items contained in each coset now become indistinguishable. For example, when we divide the integers $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ into just the even integers and the odd integers, the even integers under addition form a normal subgroup of $\mathbb{Z}$ and the resulting quotient group has just two elements that we can label "even" and "odd." In this quotient group, all the even numbers become indistinguishable from one another, and in particular, they are indistinguishable from 0 which is the identity element for the integers under addition. And similarly, the odd integers become indistinguishable from one another in this quotient group.

Now let's see what *combining* specifically has to do with quotient structures. As I've mentioned previously, in order to see a chair as a chair and not simply as a collection of component parts, our minds have to divide out whatever it is that keeps the pieces apart from one another. We have to divide out that separation by making it equivalent to 0. In other words, *combining* refers to that moment when all the pieces of the puzzle come together and we suddenly experience something totally new. This is also what I call the "aha!" moment, and this is where something-from-nothing creation really takes place. Whenever we have an epiphany that results in a new song or a new theorem or a new insight about life, we've suddenly created a new object that didn't exist before that we can now add to our table. And the crux of this type of creation is the moment in which separate pieces suddenly combine, and when whatever kept them separate is divided out from our perceptions, the result is what in mathematics we call a quotient structure. Thus, I always think of permutation groups as referring to something-from-something creation and quotient groups as referring to something-from-nothing creation.

The next step, *weighing*, refers to making it real by giving it measurable characteristics. So for instance, an author may have the intent to write a book (*decreeing*), he mulls over the plot structure (*shaping*), he has inspirations about how it will all come together (*combining*), and then he actually sits down and writes it (*weighing*).

The last step, *exchanging*, is a little harder to describe, but that is only because most of the time we tend to do that automatically. In a nutshell, if as a child we learn to see a chair as a chair in the context of a certain background, then we want to be able to still perceive that same chair even if we exchange the existing background for another. Like I said, most of us do this so automatically that this last step may not even seem worth mentioning. However, because I am now a senior citizen, I can appreciate this final stage of creation. For example, at my age both my eyes and my memory are worse, and consequently, a student that I may readily recognize in the context of the classroom, may be unrecognizable to me when that context is exchanged for another. In other words, at my age this final step of *exchanging* isn't quite as automatic as it used to be!

So, in our lives we have the opportunity to engage in these two types of creative processes. The first, something-from-something creation, refers to making permutations of what already is in order to make the world a better place. It is the process of cleaning up your life, and it is mathematically described by permutation groups. The second type of creation, something-from-nothing creation, probably happens less frequently than something-from-something creation, and it refers to process of literally creating a new world for ourselves and others through the epiphanies and insights we have. Each of these insights involves objects or ideas coming together to form a new object of experience, and this process of thoughts coming together results in what in mathematics we would call a quotient structure. The bottom line, though, is that we should all live creative lives. We should all make the world better through positive permutations of what already is, and we should all continue to grow by forming quotient structures that result in new worlds. And that's how you live a creative life.