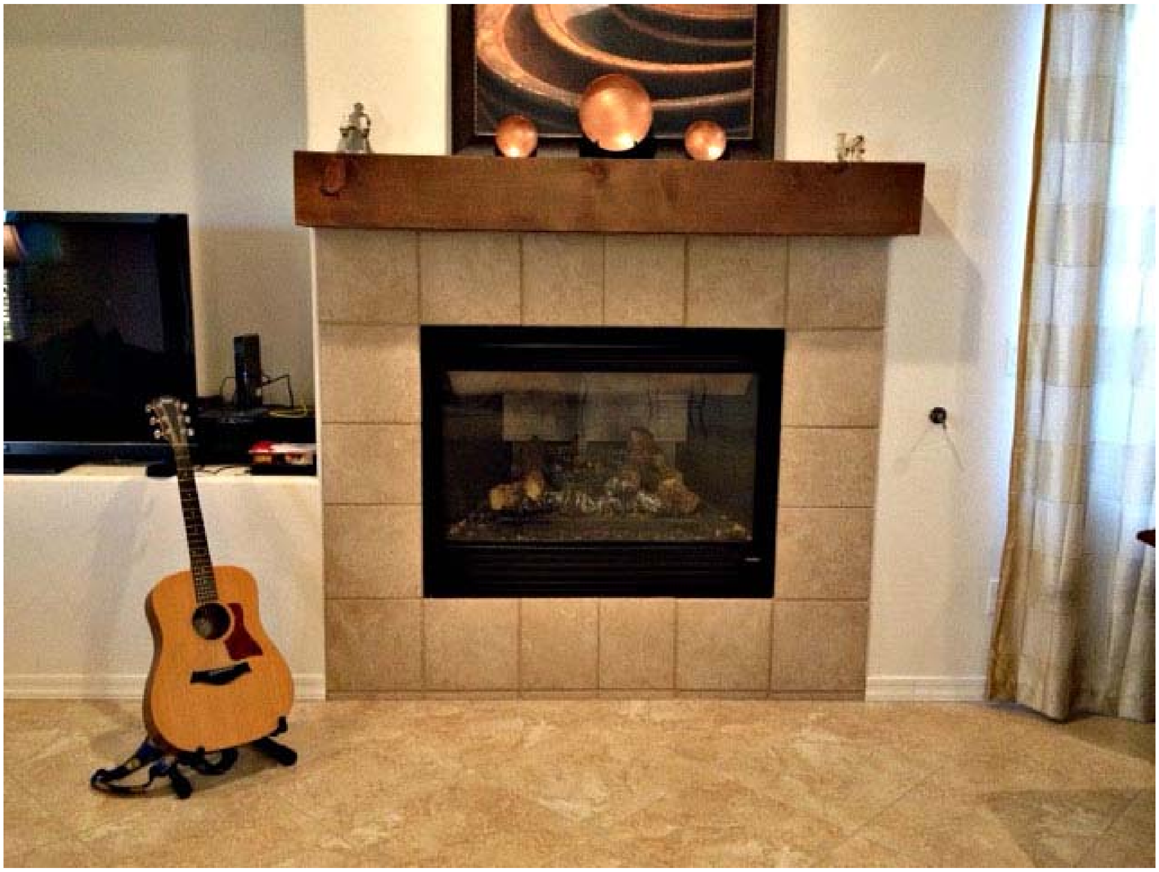
Lesson 22

FREE RANGE SYMMETRY

As we've mentioned before, groups are related to symmetry, and symmetry is just the repetition of patterns. Groups arise from these repetitions when we consider the maneuvers that are required to move a piece of a puzzle from one location to a different one that displays the same pattern. When we consider frieze groups and wallpaper groups, though, there is one very restrictive element. Namely, that our transformations be rigid ones where the entire plane is being shifted, rotated, or reflected at once. However, I often find this restriction, well, very restrictive, and so I like to indulge in what I call "free range symmetry." By this I mean simply look for patterns in your life and your environment, and don't worry about the transformations being rigid. Instead, look for symmetry and see how many different kinds of groups you might identify. And when you do this, you will gain a greater appreciation for how groups and symmetry are everywhere in our lives.

Here's an example. Below is a picture from my living room. Let's see what some of the groups are that we can identify.



Surrounding the fireplace, we see 18 tiles that are almost identical, and this brings to mind the cyclic group of order 18, $C_{18} \cong \mathbb{Z}_{18} \cong \mathbb{Z}_2 \times \mathbb{Z}_9$. We could also construct from these 18 tiles the symmetric group of degree 18, i.e. the set of all permutations of 18 objects which yields a group of size $18! \approx 6.402374 \times 10^{15}$. If we look at that tile floor and imagine those rows of square tiles extending to infinity, then the corresponding group is $\mathbb{Z} \times \mathbb{Z}$. If we now look at the front of the wooden beam above the fireplace, we see a rectangle that can be reflected about either a horizontal axis or a vertical axis. This symmetry results in the Klein 4-group, $\mathbb{Z}_2 \times \mathbb{Z}_2$. On top of the wooden beam, we see three copper bowls. As we move from one to another, we see color and shape preserved, but not size. However, if we ignore the change in size and just focus on the symmetry of shape and color, then these three bowls could suggest to us either the cyclic group of order 3 or the symmetric group of degree 3. Again, symmetry doesn't always have to mean repetitions of exact shape. Symmetry can result from repetition of colors or general shape or even patterns that we repeat over time. And finally, the guitar to the left of the fireplace exhibits bilateral symmetry, $C_2 \cong \mathbb{Z}_2$. Also, the six strings can suggest either the cyclic group of order 6, $C_6 \cong \mathbb{Z}_6$, or the symmetric group of degree 6 that contains $6! = 720$ elements. Well, these are just a few examples, but hopefully they are enough to make you realize that symmetry and groups are all around us. And this is no accident. Being able to recognize patterns is a mechanism by which our brain can efficiently organize information with a minimum of effort. For example, if you have a hundred books in your office, then it is much easier for your brain to see all of these objects as examples of just one thing, i.e. books, rather than having to try and juggle a hundred different concepts at once. Thus, patterns and symmetry help us condense a multitude of information down to a single concept, and that helps our brains more easily cope with what it perceives. Hence, symmetry is natural. Symmetry is a part of life, and we are designed to perceive it. And everywhere that there is symmetry, there are also groups. Groups are everywhere. Just go forth and let yourself be aware of that.