Factoring $x^2 + bx + c$

1. **Find integers u & v such that:**
    a. **$u + v = b$**
    b. **$uv = c$**
2. **Write $x^2 + bx + c = (x + u)(x + v)$**

Examples:

1. $x^2 + 5x + 6$

    $u = 2$
    $v = 3$

    $x^2 + 5x + 6 = (x+2)(x+3)$

2. $x^2 - x - 6$

    $u = -3$
    $v = 2$

    $x^2 - x - 6 = (x-3)(x+2)$

3. $x^2 + 8x + 15$

    $u = 3$
    $v = 5$

    $x^2 + 8x + 15 = (x+3)(x+5)$

4. $x^2 - 13x + 40$

    $u = -5$
    $v = -8$

    $x^2 - 13x + 40 = (x-5)(x-8)$

Factoring $ax^2 + bx + c$

1. **Find integers u & v such that:**
   a. **$u + v = b$**
   b. **$uv = ac$**
2. **Write $ax^2 + bx + c = ax^2 + ux + vx + c$**
3. **Factor by grouping**

Examples:

1. $2x^2 + 5x + 3$

   $u + v = 5$
   $uv = (2)(3) = 6$

   $u = 2$
   $v = 3$

   $2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3 = (2x^2 + 2x) + (3x + 3)$
   $= 2x(x + 1) + 3(x + 1) = (x+1)(2x+3)$

2. $6x^2 – x – 1$

   $u + v = -1$
   $uv = (6)(-1) = -6$

   $u = -3$
   $v = 2$

   $6x^2 – x – 1 = 6x^2 – 3x + 2x – 1 = (6x^2 – 3x) + (2x – 1)$
   $= 3x(2x – 1) + 1(2x – 1) = (2x – 1)(3x + 1)$

3. $4x^2 – 13x + 10$

   $u + v = -13$
   $uv = (4)(10) = 40$

   $u = -5$
   $v = -8$

   $4x^2 – 13x + 10 = 4x^2 – 5x – 8x + 10 = (4x^2 – 5x) + (-8x + 10)$
   $= x(4x – 5) – 2(4x – 5) = (4x – 5)(x – 2)$

Factoring $x^2 - a^2$

The difference between two perfect squares factors as $x^2 - a^2 = (x + a)(x - a)$.

Examples:

1. $x^2 - 9 = (x + 3)(x - 3)$
2. $x^2 - 25 = (x + 5)(x - 5)$
3. $9x^2 - 16 = (3x + 4)(3x - 4)$
4. $x^4 - 16 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x + 2)(x - 2)$
5. $(x + 3)^2 - 36 = ([x + 3] + 6)([x + 3] - 6) = (x + 9)(x - 3)$

Factoring $x^3 - a^3$ and $x^3 + a^3$

We can't factor the sum of two squares, but we can factor both the sum and difference of two perfect cubes. The formulas for these factorings are similar, and the proof of each is that once you multiply and the dust clears, you are left with either $x^3 + a^3$ or $x^3 - a^3$.

Formulas:

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

Examples:

1. $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$
2. $x^3 - 125 = (x - 5)(x^2 + 5x + 25)$
3. $x^3 + 1 = (x + 1)(x^2 - x + 1)$
4. $2x^3 + 16 = 2(x^3 + 8) = 2(x + 2)(x^2 - 2x + 4)$
5. $8x^3 - 27 = (2x - 3)(4x^2 + 6x + 9)$