Lesson 14

EQUIVALENCE RELATIONS – ANSWERS

Let $X = \{1,2,3,4\}$ and let
$C_2 \times C_2 \times C_2 = \{(\,),(1,2),(3,4),(5,6),(1,2)(3,4),(1,2)(5,6),(3,4)(5,6),(1,2)(3,4)(5,6)\}$. Each problem below defines and equivalence relation on either *X* or $C_2 \times C_2 \times C_2$. Find the corresponding equivalence classes.

1. Find the orbits on *X* created by $C_2 \times C_2 \times C_2$.

   The orbits in X are $\{1,2\}, \{3,4\},$ and $\{5,6\}$.

2. Partition $C_2 \times C_2 \times C_2$ into those permutations that fix 1 and those that don't.

   The sets in the partition of $C_2 \times C_2 \times C_2$ are $\{(\,),(3,4),(5,6),(3,4)(5,6)\}$ and $\{(1,2),(1,2)(3,4),(1,2)(5,6),(1,2)(3,4)(5,6)\}$.

3. Find the conjugacy classes in $C_2 \times C_2 \times C_2$.

   Since $C_2 \times C_2 \times C_2$ is abelian, every element is in its own conjugacy class. Thus, the conjugacy classes in $C_2 \times C_2 \times C_2$ are $\{(\,)\}, \{(1,2)\}, \{(3,4)\}, \{(5,6)\}, \{(1,2)(3,4)\}, \{(1,2)(5,6)\}, \{(3,4)(5,6)\},$ and $\{(1,2)(3,4)(5,6)\}$.

4. <u>BONUS PROBLEM:</u> If $X = \{1,2,3,4\}$ and $D_4 = \{(\,),(1,3),(2,4),(1,3)(2,4),(1,2)(3,4),(1,4)(2,3),(1,2,3,4),(1,4,3,2)\}$, find the five conjugacy classes in D_4.

   $\{(\,)\}$
   $\{(1,3),(2,4)\}$
   $\{(1,2)(3,4),(1,4)(2,3)\}$
   $\{(1,2,3,4),(1,4,3,2)\}$
   $\{(1,3)(2,4)\}$

   Notice that conjugate elements have the same cycle structure. Important!