

Lesson 5

DIRECT PRODUCTS – PRACTICE

1. Below is a multiplication table for D_3 where R represents a rotation and F represents a flip of an equilateral triangle.

	e	R	R^2	F	FR	FR^2
e	e	R	R^2	F	FR	FR^2
R	R	R^2	e	FR^2	F	FR
R^2	R^2	e	R	FR	FR^2	F
F	F	FR	FR^2	e	R	R^2
FR	FR	FR^2	F	R^2	e	R
FR^2	FR^2	F	FR	R	R^2	e

- a. How many elements are in $\mathbb{Z}_2 \times D_3$?
 - b. List in coordinate form the elements in $\mathbb{Z}_2 \times D_3$.
 - c. Is $\mathbb{Z}_2 \times D_3$ abelian? If not, then give two elements that do not commute with one another along with their products.
2. What two cyclic groups can we write \mathbb{Z}_{10} as a product of?
 3. With \mathbb{Z}_{10} expressed as a product of two cyclic groups, list the elements in \mathbb{Z}_{10} in coordinate form.
 4. Using your answer to the previous problem, find an element that generates \mathbb{Z}_{10} .
 5. What is the order of $\mathbb{Z}_3 \times \mathbb{Z}_3$?
 6. What is the order of every non-identity element in $\mathbb{Z}_3 \times \mathbb{Z}_3$? Conclude that $\mathbb{Z}_3 \times \mathbb{Z}_3$ is not isomorphic to \mathbb{Z}_9 since it has no element of order nine.