Lesson 5

DIRECT PRODUCTS – ANSWERS

1. Below is a multiplication table for D_3 where R represents a rotation and F represents a flip of an equilateral triangle.

	e	R	R^2	F	FR	FR^2
e	e	R	R^2	F	FR	FR^2
R	R	R^2	e	FR^2	F	FR
R^2	R^2	e	R	FR	FR^2	F
F	F	FR	FR^2	e	R	R^2
FR	FR	FR^2	F	R^2	e	R
FR^2	FR^2	F	FR	R	R^2	e

a. How many elements are in $\mathbb{Z}_2 \times D_3$?

$|\mathbb{Z}_2 \times D_3| = |\mathbb{Z}_2| \cdot |D_3| = 2 \cdot 6 = 12$

b. List in coordinate form the elements in $\mathbb{Z}_2 \times D_3$.

$$\mathbb{Z}_2 \times D_3 = \begin{Bmatrix} (0,e),(0,R),(0,R^2),(0,F),(0,FR),(0,FR^2), \\ (1,e),(1,R),(1,R^2),(1,F),(1,FR),(1,FR^2) \end{Bmatrix}$$

c. Is $\mathbb{Z}_2 \times D_3$ abelian? If not, then give two elements that do not commute with one another along with their products.

No, it's not abelian since $(0,R)*(0,F)=(0,FR^2)$, but $(0,F)*(0,R)=(0,FR)$.

2. What two cyclic groups can we write $\mathbb{Z}_{10}$ as a product of?

$\mathbb{Z}_{10} \cong \mathbb{Z}_2 \times \mathbb{Z}_5$

3. With $\mathbb{Z}_{10}$ expressed as a product of two cyclic groups, list the elements in $\mathbb{Z}_{10}$ in coordinate form.

$\mathbb{Z}_{10} \cong \mathbb{Z}_2 \times \mathbb{Z}_5 = \{(0,0),(0,1),(0,2),(0,3),(0,4),(1,0),(1,1),(1,2),(1,3),(1,4)\}$

4. Using your answer to the previous problem, find an element that generates $\mathbb{Z}_{10}$.

$$(1,1)=(1,1)$$
$$(1,1)+(1,1)=(0,2)$$
$$(1,1)+(1,1)+(1,1)=(1,3)$$
$$(1,1)+(1,1)+(1,1)+(1,1)=(0,4)$$
$$(1,1)+(1,1)+(1,1)+(1,1)+(1,1)=(1,0)$$
$$(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)=(0,1)$$
$$(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)=(1,2)$$
$$(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)=(0,3)$$
$$(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)=(1,4)$$
$$(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)+(1,1)=(0,0)$$

Therefore, $\langle(1,1)\rangle \cong \mathbb{Z}_{10}$

5. What is the order of $\mathbb{Z}_3 \times \mathbb{Z}_3$?

$$|\mathbb{Z}_2 \times \mathbb{Z}_3| = |\mathbb{Z}_2| \cdot |\mathbb{Z}_3| = 3 \cdot 3 = 9$$

6. What is the order of every non-identity element in $\mathbb{Z}_3 \times \mathbb{Z}_3$? Conclude that $\mathbb{Z}_3 \times \mathbb{Z}_3$ is not isomorphic to $\mathbb{Z}_9$ since it has no element of order nine.

Every non-identity element in $\mathbb{Z}_3 \times \mathbb{Z}_3$ has order 3. Therefore, it can't be isomorphic to $\mathbb{Z}_9$ which has an element of order 9.