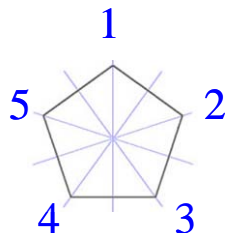


Lesson 3

DIHEDRAL GROUPS – ANSWERS

Consider the regular polygon below.



1. Find $|D_5|$.

$$|D_5| = 10$$

2. Find the elements in D_5 . List these elements as permutations.

$$D_5 = \left\{ \begin{array}{l} () , (1, 2, 3, 4, 5), (1, 3, 5, 2, 4), (1, 4, 2, 5, 3), (1, 5, 4, 3, 2), \\ (2, 5)(3, 4), (1, 3)(4, 5), (2, 4)(1, 5), (1, 2)(3, 5), (1, 4)(2, 3) \end{array} \right\}$$

3. What are the possible orders for subgroups of D_5 ?

1, 2, 5, or 10

4. The dihedral group D_5 has 8 subgroups. Find all the subgroups of D_5 .

$$\langle () \rangle = \{ () \}$$

$$\langle (1, 2, 3, 4, 5) \rangle = \{ (), (1, 2, 3, 4, 5), (1, 3, 5, 2, 4), (1, 4, 2, 5, 3), (1, 5, 4, 3, 2) \}$$

$$\langle (1, 3, 5, 2, 4) \rangle = \{ (), (1, 2, 3, 4, 5), (1, 3, 5, 2, 4), (1, 4, 2, 5, 3), (1, 5, 4, 3, 2) \}$$

$$\langle (1, 4, 2, 5, 3) \rangle = \{ (), (1, 2, 3, 4, 5), (1, 3, 5, 2, 4), (1, 4, 2, 5, 3), (1, 5, 4, 3, 2) \}$$

$$\langle (1, 5, 4, 3, 2) \rangle = \{ (), (1, 2, 3, 4, 5), (1, 3, 5, 2, 4), (1, 4, 2, 5, 3), (1, 5, 4, 3, 2) \}$$

$$\langle (2, 5)(3, 4) \rangle = \{ (), (2, 5)(3, 4) \}$$

$$\langle (1, 3)(4, 5) \rangle = \{ (), (1, 3)(4, 5) \}$$

$$\langle (2, 4)(1, 5) \rangle = \{ (), (2, 4)(1, 5) \}$$

$$\langle (1, 2)(3, 5) \rangle = \{ (1, 2)(3, 5) \}$$

$$\langle (1, 4)(2, 3) \rangle = \{ (1, 4)(2, 3) \}$$

$$D_5 = \langle (1, 2, 3, 4, 5), (2, 5)(3, 4) \rangle \left\{ \begin{array}{l} () , (1, 2, 3, 4, 5), (1, 3, 5, 2, 4), (1, 4, 2, 5, 3), (1, 5, 4, 3, 2), \\ (2, 5)(3, 4), (1, 3)(4, 5), (2, 4)(1, 5), (1, 2)(3, 5), (1, 4)(2, 3) \end{array} \right\}$$

Lesson 3

In the list above, we have 7 distinct cyclic groups plus the entire group D_5 , and that make eight subgroups in all. Thus, all eight subgroups of D_5 are accounted for.