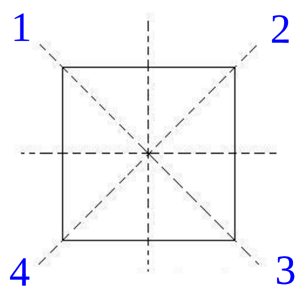
Lesson 2

CYCLIC GROUPS AN SUBGROUPS – ANSWERS

Recall that f you rotate the square below through multiples of $90°$ or flip the square about one of the indicated axes of symmetry, then eight configurations are possible. These eight configurations give rise to the following group.

$$G = \{(), (1,2,3,4), (1,3)(2,4), (1,4,3,2), (1,2)(3,4), (1,4)(2,3), (1,3), (2,4)\}$$

For each element in this group, find the corresponding cyclic group generated and give the order of that subgroup. Is *G*, itself, a cyclic group? Also, find two elements which commute with one another and show the results of multiplying the elements together in each of the two possible orders. Now repeat, but this time find two elements that don't commute with one another. And finally find a product involving rotations and the flip $(1,3)$ that will give the result $(2,4)$. The rotations that may be used are $(1,2,3,4), (1,3)(2,4),$ and/or $(1,4,3,2)$.



1. The cyclic groups and their orders are as follows.

$$\langle () \rangle = \{()\}, \ |\langle () \rangle| = 1$$

$$\langle (1,2,3,4) \rangle = \{(), (1,2,3,4), (1,3)(2,4), (1,4,3,2)\}, \ |\langle (1,2,3,4) \rangle| = 4$$

$$\langle (1,3)(2,4) \rangle = \{(), (1,3)(2,4)\}, \ |\langle (1,3)(2,4) \rangle| = 2$$

$$\langle (1,4,3,2) \rangle = \{(), (1,4,3,2), (1,3)(2,4), (1,2,3,4)\}, \ |\langle (1,4,3,2) \rangle| = 4$$

$$\langle (1,2)(3,4) \rangle = \{(), (1,2)(3,4)\}, \ |\langle (1,2)(3,4) \rangle| = 2$$

$$\langle (1,4)(2,3) \rangle = \{(), (1,4)(2,3)\}, \ |\langle (1,4)(2,3) \rangle| = 2$$

$$\langle (1,3) \rangle = \{(), (1,3)\}, \ |\langle (1,3) \rangle| = 2$$

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$\langle (2,4) \rangle = \{ (), (2,4) \}$, $|\langle (2,4) \rangle| = 2$

2. G is not cyclic since it is not generated by a single element contained in the group.

3. $(1,2,3,4)(1,4,3,2) = ()$ and $(1,4,3,2)(1,2,3,4) = ()$.

4. $(1,2,3,4)(1,3) = (1,2)(3,4)$ and $(1,3)(1,2,3,4) = (1,4)(2,3)$.

5. $[(1,3)(2,4)] \cdot (1,3) = (1,3)(2,4)(1,3) = (2,4)$