

# Lesson 15

## COSETS – ANSWERS

The symmetric group of degree 4 is

$$S_4 = \left\{ \begin{array}{l} ( ), (1,2), (1,3), (1,4), (2,3), (2,4), (3,4), (1,2)(3,4), (1,3)(2,4), (1,4)(2,3), (1,2,3), \\ (1,3,2), (1,2,4), (1,4,2), (1,4,3), (2,3,4), (2,4,3), (1,2,3,4), ((1,2,4,3), (1,3,4,3), (1,3,2,4), \\ (1,4,3,2), (1,4,2,3) \end{array} \right\}.$$

For the given subgroup  $H$ , find the total number of left or right cosets of  $H$  in  $S_4$ , and find the specific left and right cosets of  $H$  in  $S_4$  created by the permutation  $(1,2)$ .

- $H = \{ ( ), (2,4), (1,2)(3,4), (1,2,3,4), (1,3), (1,3)(2,4), (1,4,3,2), (1,4)(2,3) \}$ . (NOTE:  $H$  is not normal (self-conjugate) in  $S_4$ .)

The number of cosets of  $H$  in  $S_4$  is  $\frac{|S_4|}{|H|} = \frac{24}{8} = 3$ .

$$(1,2)H = (1,2) \left\{ \begin{array}{l} ( ) \\ (2,4) \\ (1,2)(3,4) \\ (1,2,3,4) \\ (1,3) \\ (1,3)(2,4) \\ (1,4,3,2) \\ (1,4)(2,3) \end{array} \right\} = \left\{ \begin{array}{l} (1,2)( ) \\ (1,2)(2,4) \\ (1,2)(1,2)(3,4) \\ (1,2)(1,2,3,4) \\ (1,2)(1,3) \\ (1,2)(1,3)(2,4) \\ (1,2)(1,4,3,2) \\ (1,2)(1,4)(2,3) \end{array} \right\} = \left\{ \begin{array}{l} (1,2) \\ (1,4,2) \\ (3,4) \\ (3,4) \\ (1,2,3) \\ (1,4,2,3) \\ (2,4,3) \\ (1,3,2,4) \end{array} \right\}$$

$$H(1,2) = \left\{ \begin{array}{l} ( ) \\ (2,4) \\ (1,2)(3,4) \\ (1,2,3,4) \\ (1,3) \\ (1,3)(2,4) \\ (1,4,3,2) \\ (1,4)(2,3) \end{array} \right\} (1,2) = \left\{ \begin{array}{l} ( )(1,2) \\ (2,4)(1,2) \\ (1,2)(3,4)(1,2) \\ (1,2,3,4)(1,2) \\ (1,3)(1,2) \\ (1,3)(2,4)(1,2) \\ (1,4,3,2)(1,2) \\ (1,4)(2,3)(1,2) \end{array} \right\} = \left\{ \begin{array}{l} (1,2) \\ (1,2,4) \\ (3,4) \\ (2,3,4) \\ (1,3,2) \\ (1,3,2,4) \\ (1,4,3) \\ (1,4,2,3) \end{array} \right\}$$

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$$2. \quad H = \left\{ ( ), (2,3,4), (2,4,3), (1,2)(3,4), (1,2,3), (1,2,4), (1,3,2), (1,3,4), (1,3)(2,4), (1,4,2), \right. \\ \left. (1,4,3), (1,4)(2,3) \right\}.$$

(NOTE:  $H$  is normal (self-conjugate) in  $S_4$ .)

The number of cosets of  $H$  in  $S_4$  is  $\frac{|S_4|}{|H|} = \frac{24}{12} = 2$ .

$$(1,2)H = (1,2) \left\{ \begin{array}{l} ( ) \\ (2,3,4) \\ (2,4,3) \\ (1,2)(3,4) \\ (1,2,3) \\ (1,2,4) \\ (1,3,2) \\ (1,3,4) \\ (1,3)(2,4) \\ (1,4,2) \\ (1,4,3) \\ (1,4)(2,3) \end{array} \right\} = \left\{ \begin{array}{l} (1,2)( ) \\ (1,2)(2,3,4) \\ (1,2)(2,4,3) \\ (1,2)(1,2)(3,4) \\ (1,2)(1,2,3) \\ (1,2)(1,2,4) \\ (1,2)(1,3,2) \\ (1,2)(1,3,4) \\ (1,2)(1,3)(2,4) \\ (1,2)(1,4,2) \\ (1,2)(1,4,3) \\ (1,2)(1,4)(2,3) \end{array} \right\} = \left\{ \begin{array}{l} (1,2) \\ (1,3,4,2) \\ (1,4,3,2) \\ (3,4) \\ (1,3) \\ (1,4) \\ (2,3) \\ (1,2,3,4) \\ (1,4,2,3) \\ (2,4) \\ (1,2,4,3) \\ (1,3,2,4) \end{array} \right\}$$

$$H(1,2) = \left\{ \begin{array}{l} ( ) \\ (2,3,4) \\ (2,4,3) \\ (1,2)(3,4) \\ (1,2,3) \\ (1,2,4) \\ (1,3,2) \\ (1,3,4) \\ (1,3)(2,4) \\ (1,4,2) \\ (1,4,3) \\ (1,4)(2,3) \end{array} \right\} (1,2) = \left\{ \begin{array}{l} ( )(1,2) \\ (2,3,4)(1,2) \\ (2,4,3)(1,2) \\ (1,2)(3,4)(1,2) \\ (1,2,3)(1,2) \\ (1,2,4)(1,2) \\ (1,3,2)(1,2) \\ (1,3,4)(1,2) \\ (1,3)(2,4)(1,2) \\ (1,4,2)(1,2) \\ (1,4,3)(1,2) \\ (1,4)(2,3)(1,2) \end{array} \right\} = \left\{ \begin{array}{l} (1,2) \\ (1,2,3,4) \\ (1,2,4,3) \\ (3,4) \\ (2,3) \\ (2,4) \\ (1,3) \\ (1,3,4,2) \\ (1,3,2,4) \\ (1,4) \\ (1,4,3,2) \\ (1,4,2,3) \end{array} \right\}$$