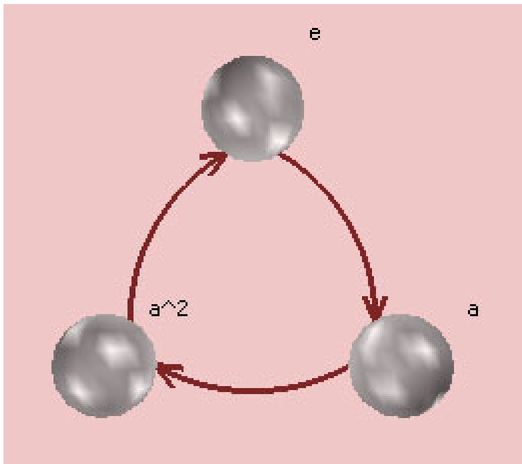
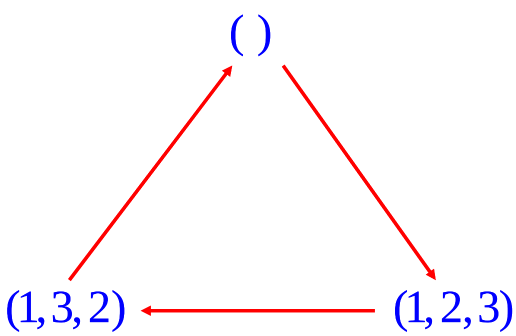
Lesson 8

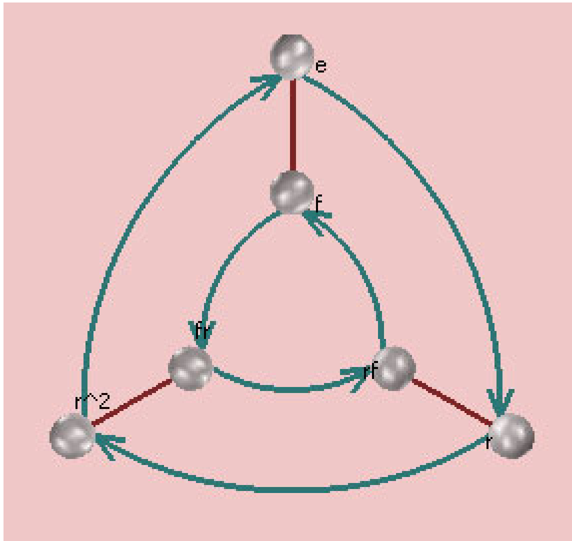
CAYLEY GRAPHS

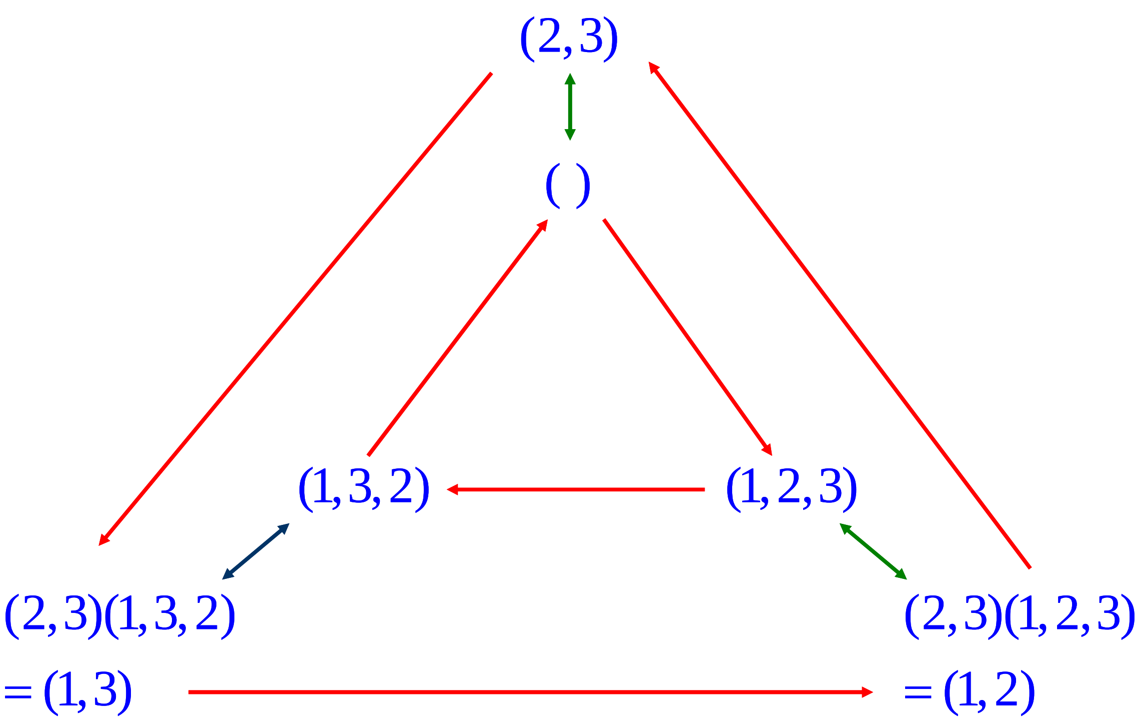
Another way of representing a group visually that is very popular is by means of a Cayley graph. This is a method pioneered by the British mathematician Arthur Cayley (1821-1895). In this method, we begin with a set of generators for a group and start at the identity. From the identity, we apply each generator in order to arrive at additional elements in the group. And then once we get to those elements, multiply by our generators again to create even more elements. And when we are done, all elements in the group should be accounted for, and every element should have an arrow for each generator both arriving at that element and leaving that element. Here are a few examples beginning with the cyclic group C_3. I'll first show a rather traditional Cayley graph for this group and then mine using cycle notation.



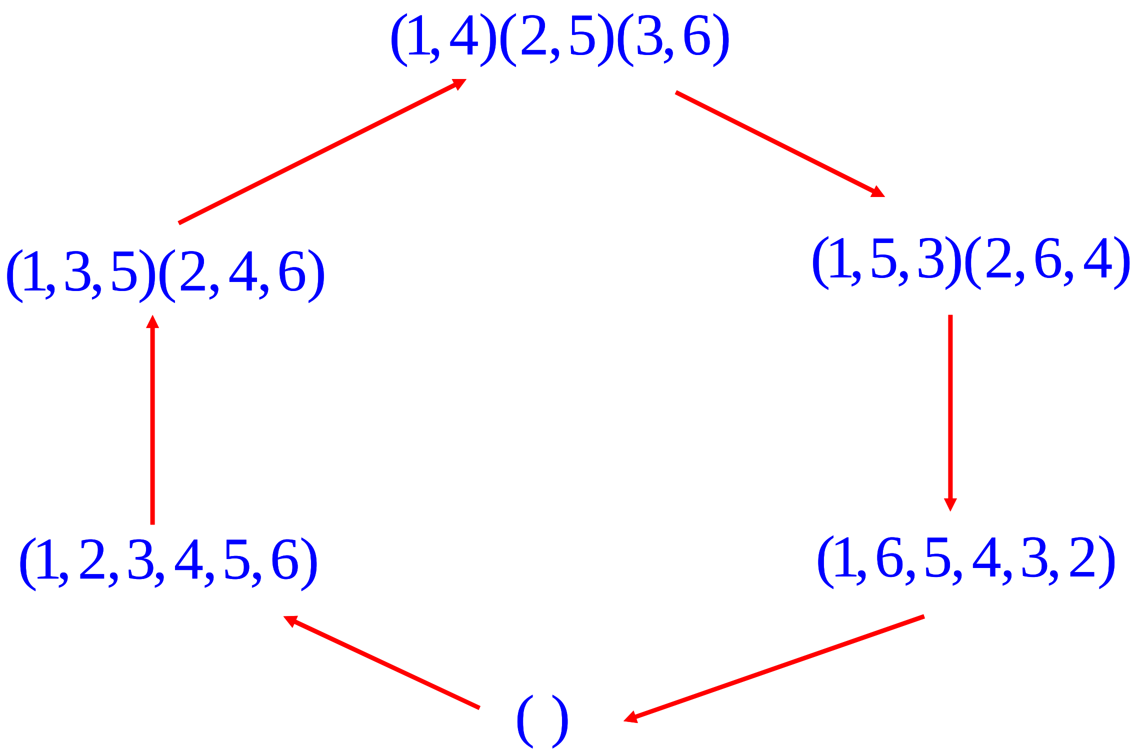


So far this doesn't look particularly different from other diagrams we've explored, but once we get into a group generated by two elements, then we'll immediately see how it differs. Thus, let's examine the dihedral group D_3 and look at a couple of different versions of its Cayley graph. Also, notice that in our version we use different colored arrows for each generator. In particular, we use a red arrow for multiplication on the left by $(1,2,3)$, and we use a green arrow for multiplication on the left by $(2,3)$.

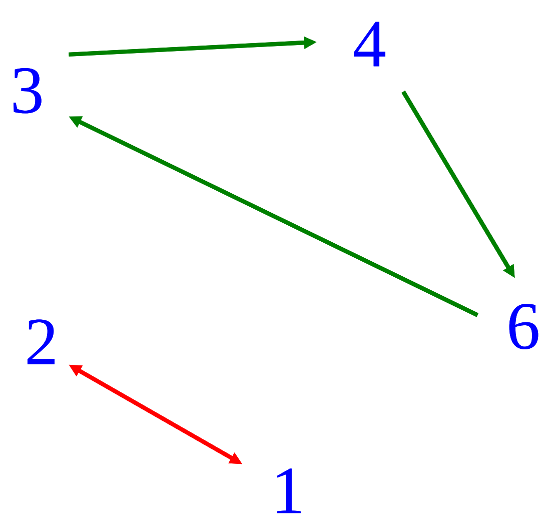


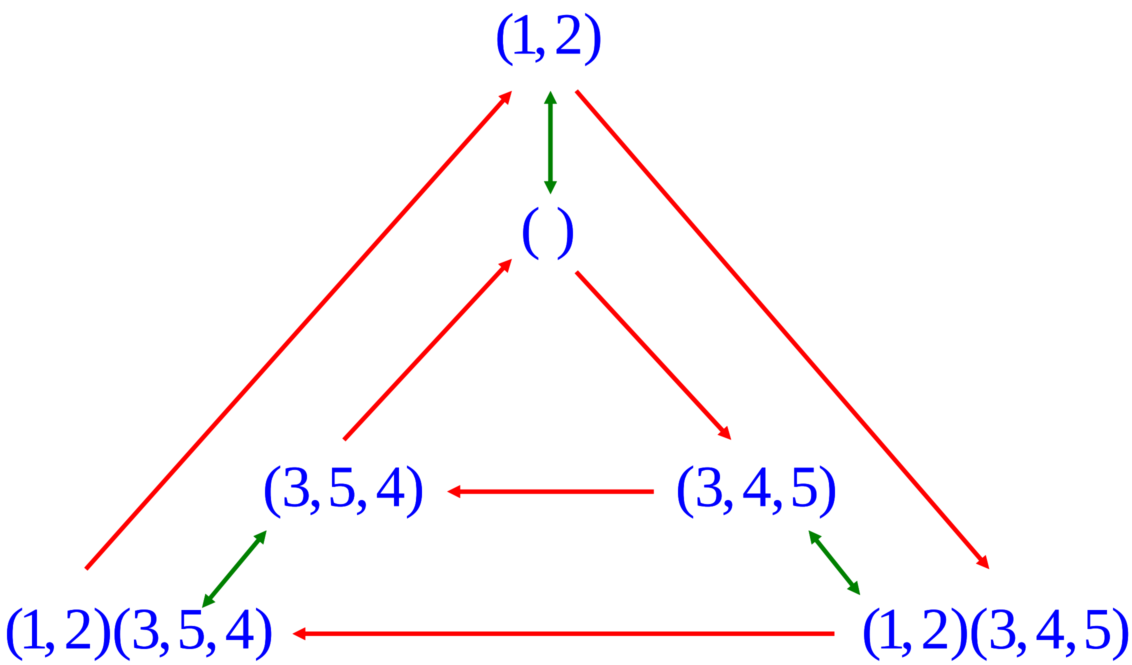


Now let's look at a Cayley graph for C_6 where the generator is $(1,2,3,4,5,6)$. As you might expect, the structure is pretty simple.



However, recall that C_6 can also be written as the direct product of C_2 and C_3, $C_2 \times C_3$. This representation results in the following generator diagram showing two generators, $(1,2)$ & $(3,4,5)$, and a far more intricate Cayley graph.





And finally, notice that this Cayley graph for C_6 (above) looks a lot like our Cayley graph for D_3 (below) with the exception that our red arrows are always oriented in the clockwise direction. In contrast, the Cayley graph for D_3 has one set of red arrows oriented clockwise and the other oriented counterclockwise.