

Lesson 8

CAYLEY GRAPHS – PRACTICE

As we've mentioned before, there are five distinct, non-isomorphic groups of order 8. Each problem below lists the elements in one of these groups of order 8 along with a set of generators. Use the generators to construct the corresponding Cayley graph.

1. $C_8 = \{ (), (1,2,3,4,5,6,7,8), (1,3,5,7)(2,4,6,8), (1,4,7,2,5,8,3,6), (1,5)(2,6)(3,7)(4,8), (1,6,3,8,5,2,7,4), (1,7,5,3)(2,8,6,4), (1,8,7,6,5,4,3,2) \}$

$$\text{Generators} = \{(1,2,3,4,5,6,7,8)\}$$

2. $D_4 = \{ (), (2,4), (1,2)(3,4), (1,2,3,4), (1,3), (1,3)(2,4), (1,4,3,2), (1,4)(2,3) \}$

$$\text{Generators} = \{(1,2,3,4), (1,2)(3,4)\}$$

3. $C_2 \times C_4 = \{ (), (3,4,5,6), (3,5)(4,6), (3,6,5,4), (1,2), (1,2)(3,4,5,6), (1,2)(3,5)(4,6), (1,2)(3,6,5,4) \}$

$$\text{Generators} = \{(1,2), (3,4,5,6)\}$$

4. $C_2 \times C_2 \times C_2 = \{ (), (5,6), (3,4), (3,4)(5,6), (1,2), (1,2)(5,6), (1,2)(3,4), (1,2)(3,4)(5,6) \}$

$$\text{Generators} = \{(1,2), (3,4)(5,6)\}$$

5. $Q = \{ (), (1,2,5,6)(3,8,7,4), (1,3,5,7)(2,4,6,8), (1,4,5,8)(2,7,6,3), (1,5)(2,6)(3,7)(4,8), (1,6,5,2)(3,4,7,8), (1,7,5,3)(2,8,6,4), (1,8,5,4)(2,3,6,7) \}$

$$\text{Generators} = \{(1,2,5,6)(3,8,7,4), (1,4,5,8)(2,7,6,3)\}$$