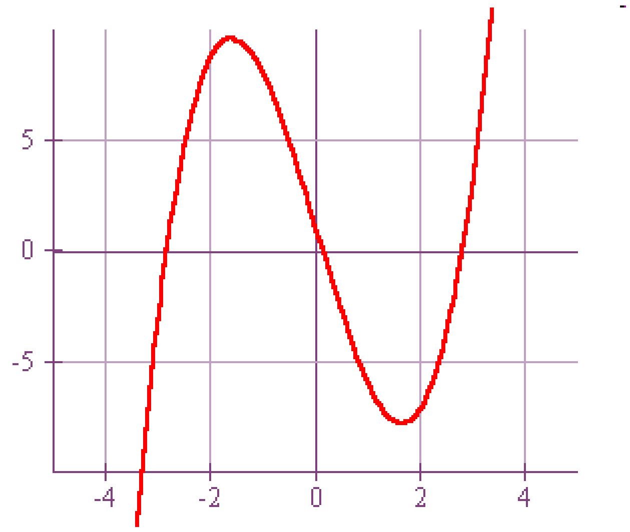
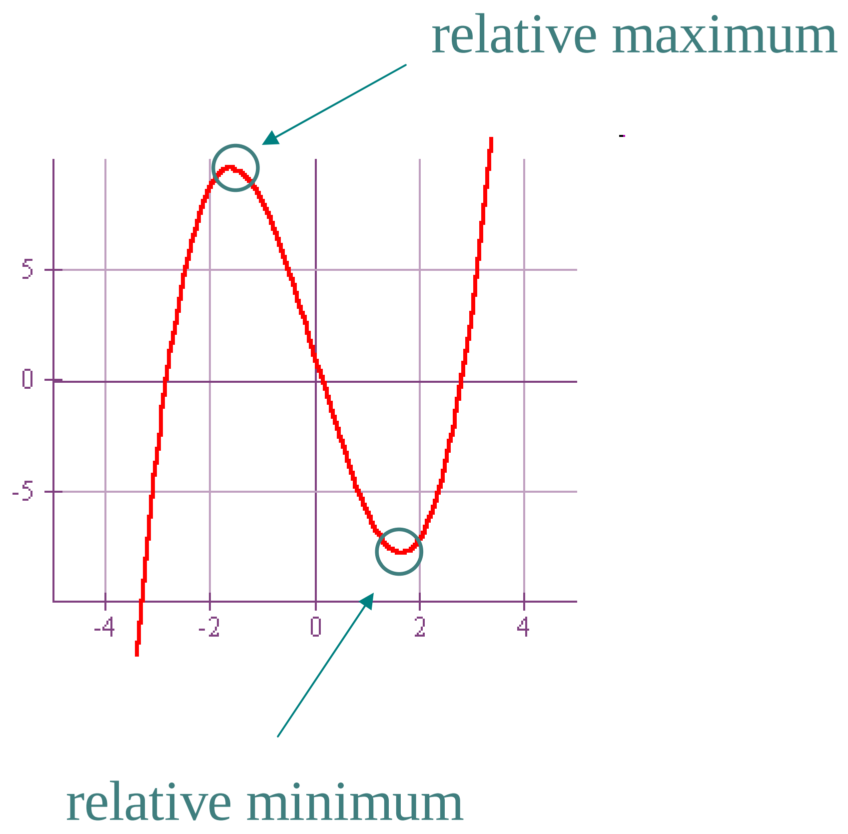
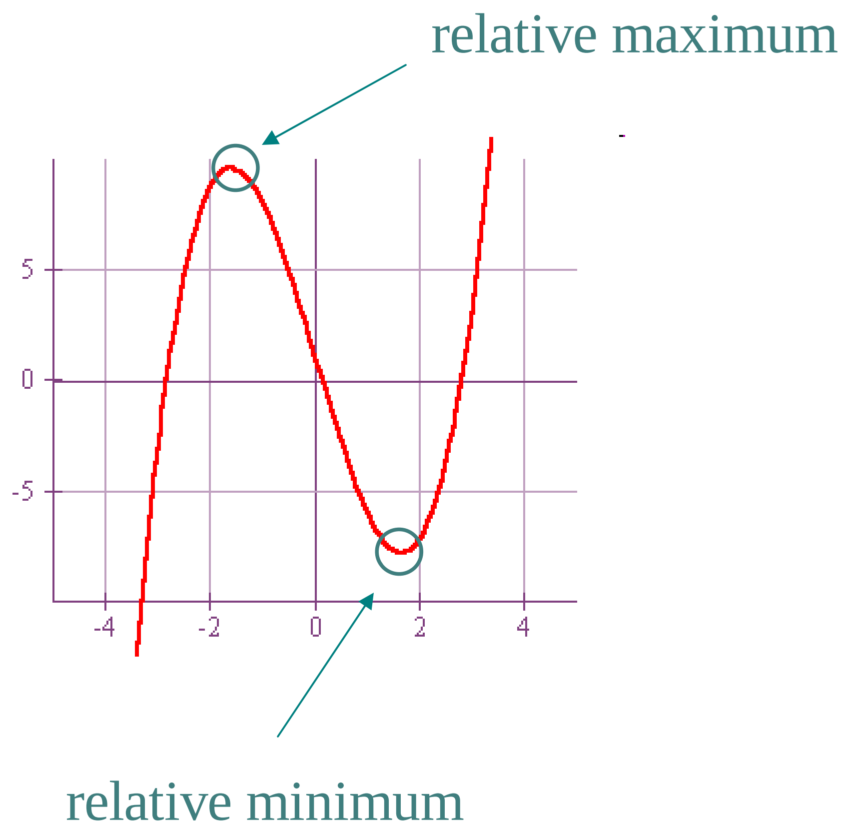
RELATIVE EXTREMA



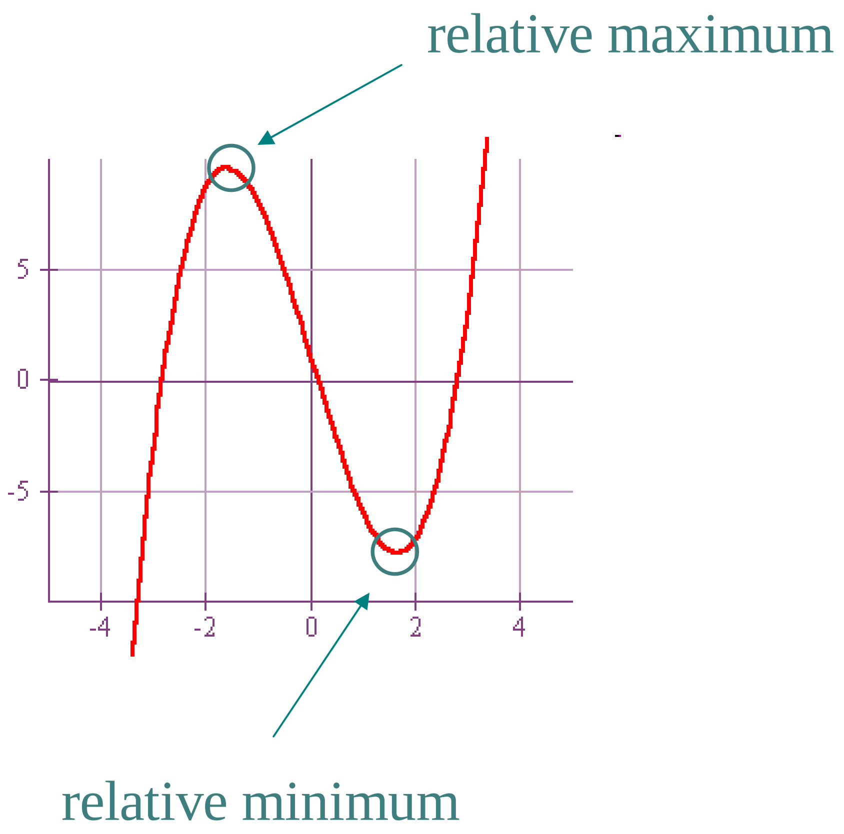
We often think of a *relative* or *local maximum* as a point that is at the top of a hill and a *relative* or *local minimum* as a point that is at the bottom of a valley.



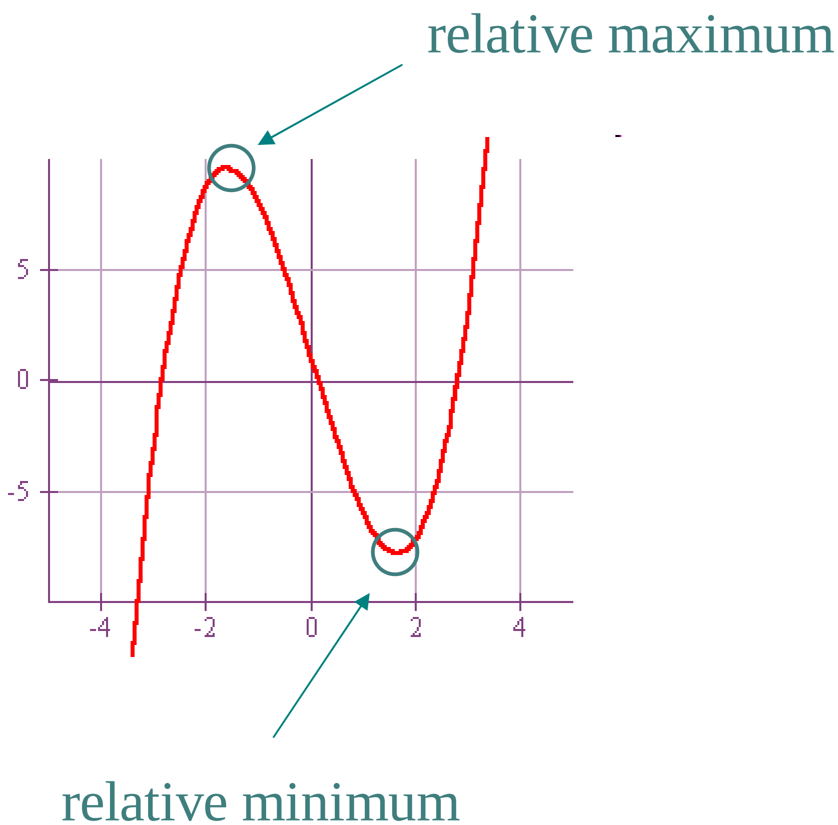
The function value at a *relative maximum* is greater than or equal to that of points close by.



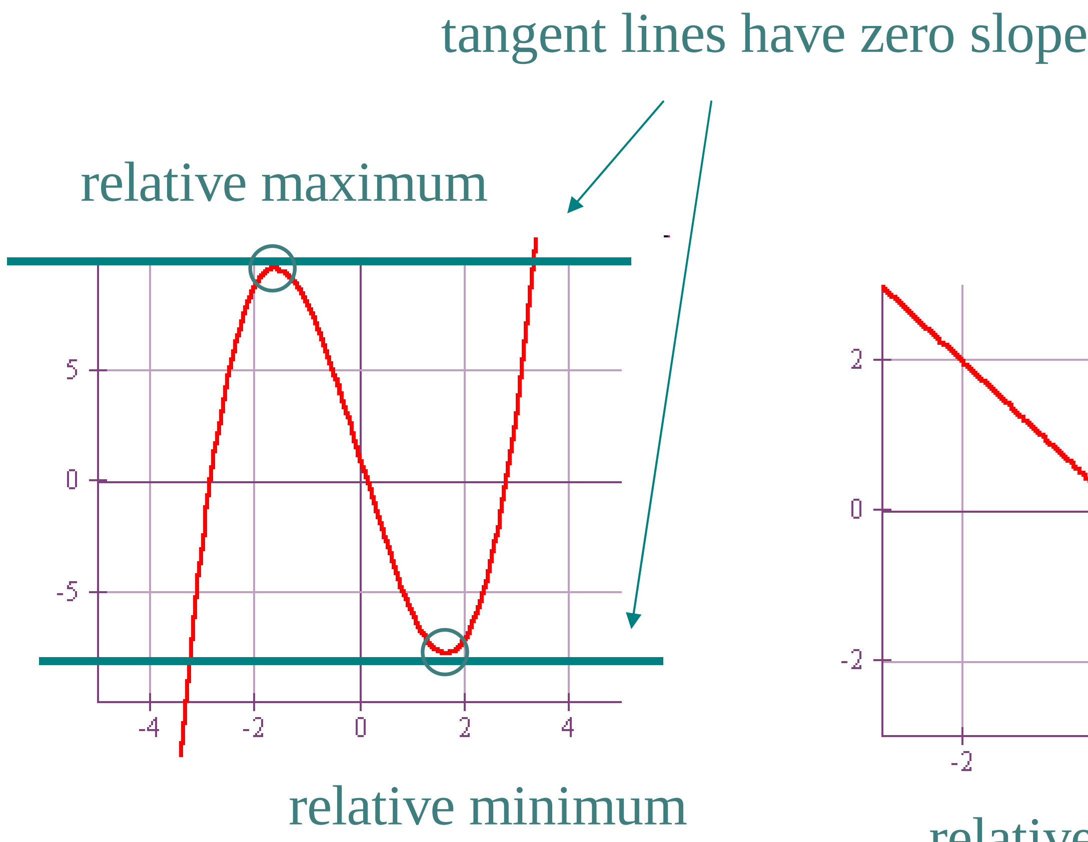
The function value at a *relative minimum* is less than or equal to that of points close by.

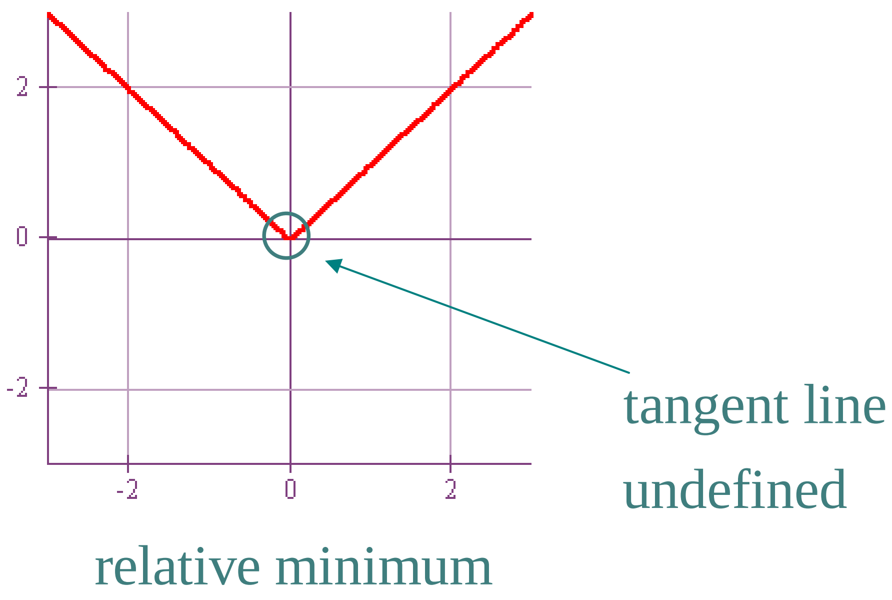


Together, we call these points *relative extrema*.

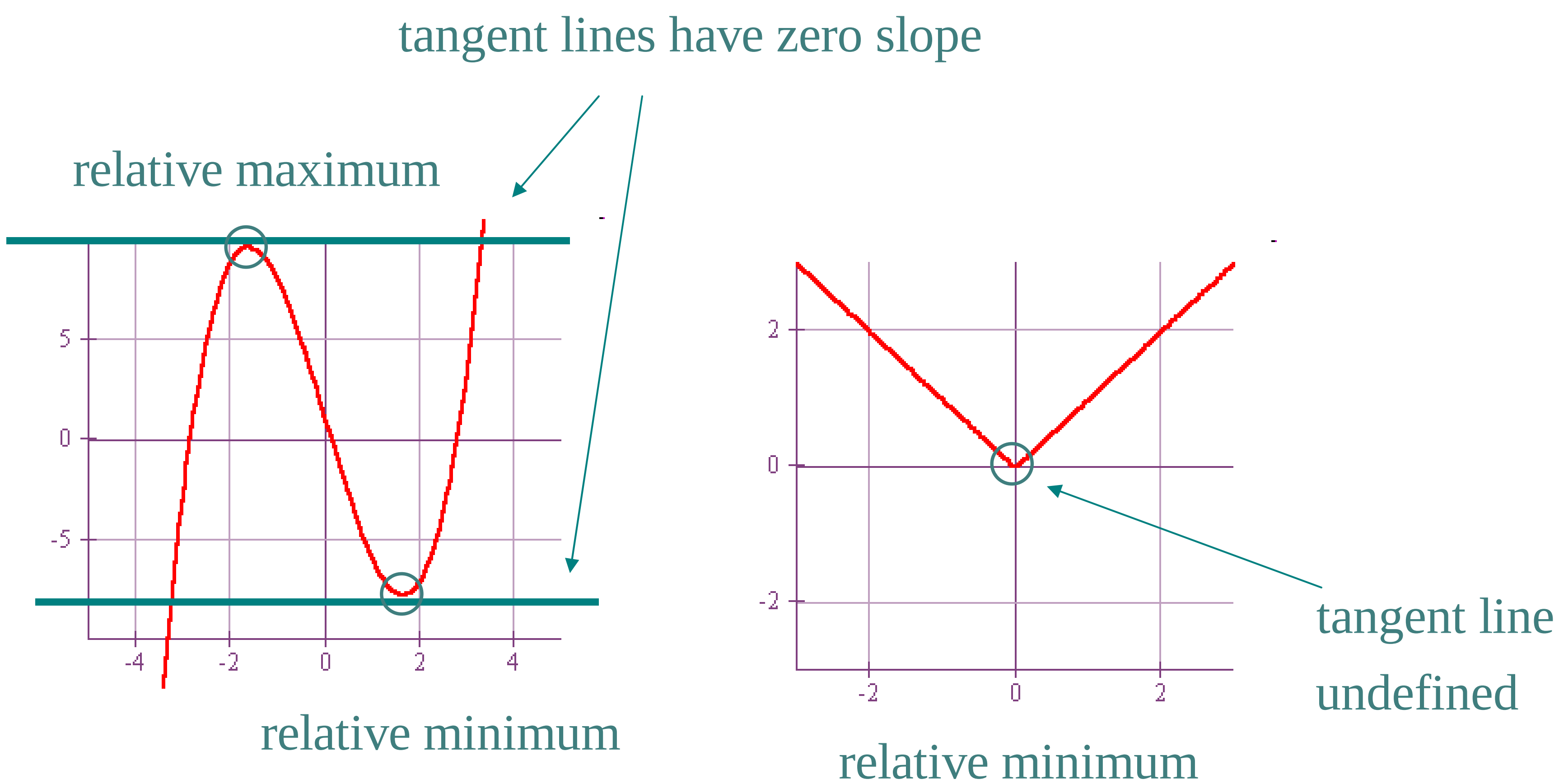


A *relative extreme value* can only occur at a point where the derivative is zero or where it is undefined.

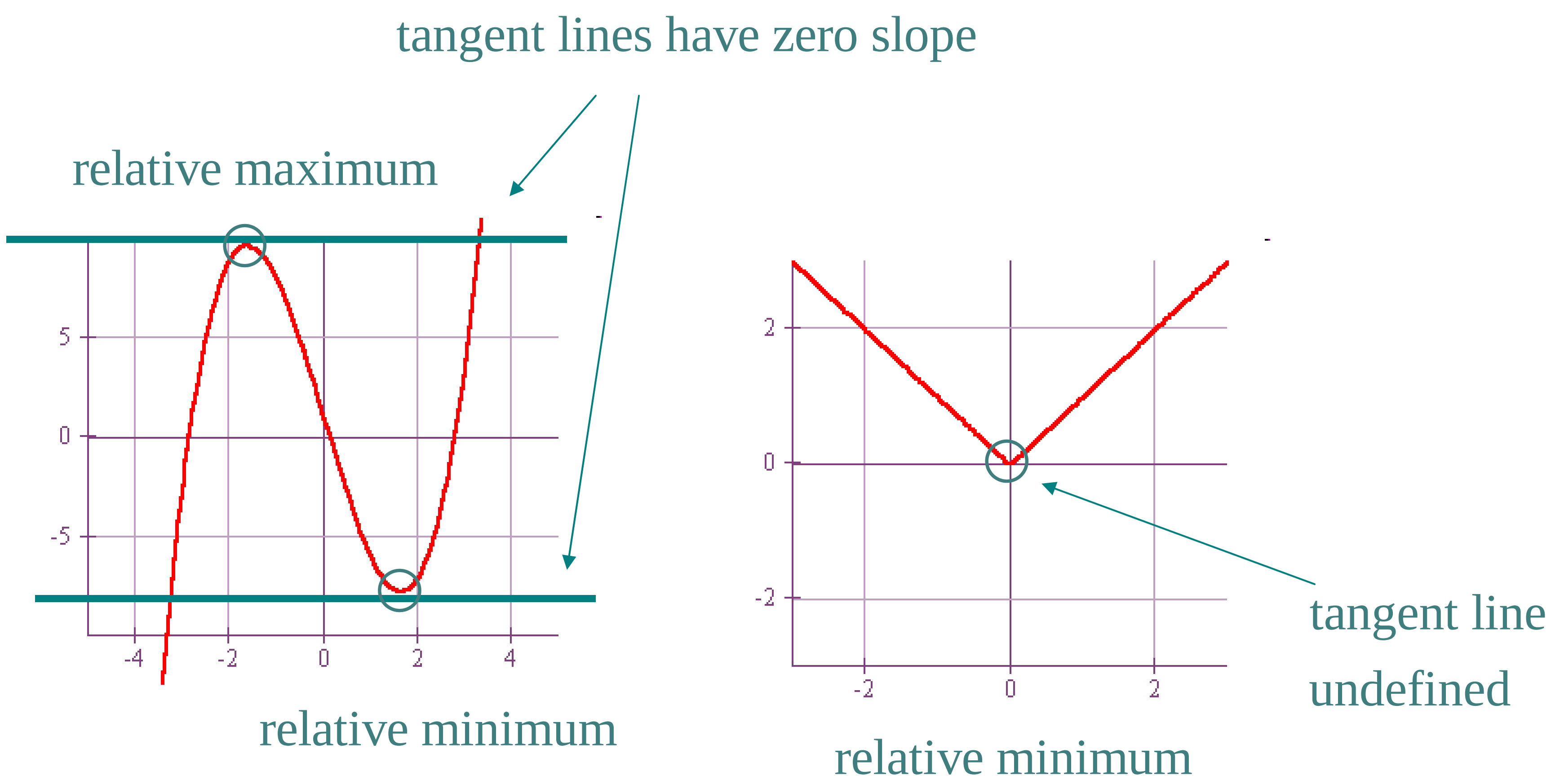




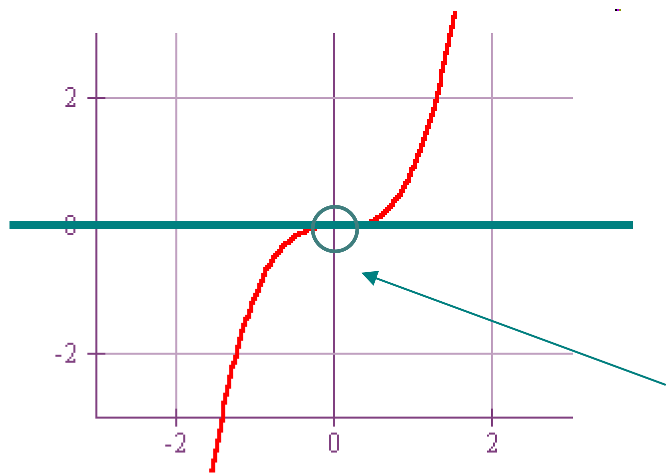
A point where the first derivative is zero or undefined is called a *critical point*.



Relative maximums and *minimums* always occur at *critical points*.



On the other hand, you can have a *critical point* without it being either a *relative maximum* or *minimum*.

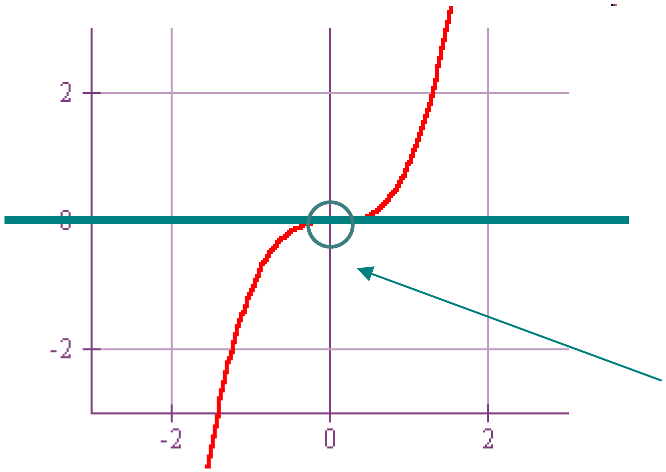


$$f(x) = x^3$$

critical point, but neither a maximum not a minimum

To find a *relative maximum* or *minimum*:

1. Find all the critical points.
2. Examine the graph to see if you have a relative max or min.



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critical point, but neither a maximum not a minimum

EXAMPLE: Find the relative extrema for $f(x) = x^2 + 2x - 3$.

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$$f'(x) = 0 \Rightarrow 2x + 2 = 0 \Rightarrow x = -1$$

critical point

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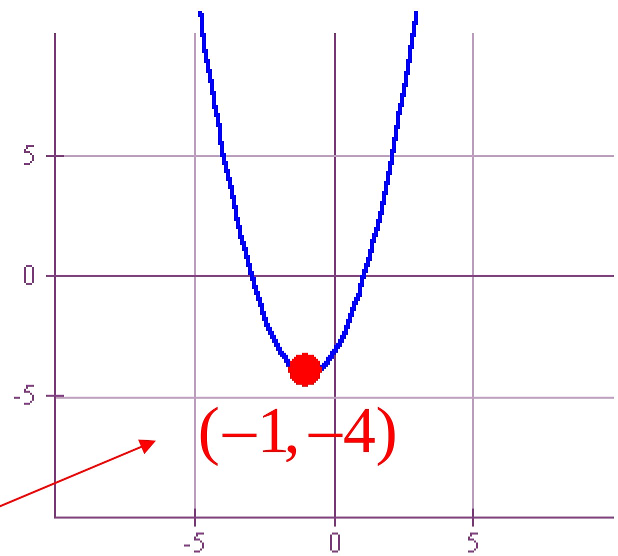
$$f(-1) = 1 - 2 - 3 = -4$$

relative minimum



relative minimum point

Another way to khow that we have a relative minimum is to observe that the sign of the derivative changes from negative to positive as we pass the critical point. This is known as the First Derivative Test.

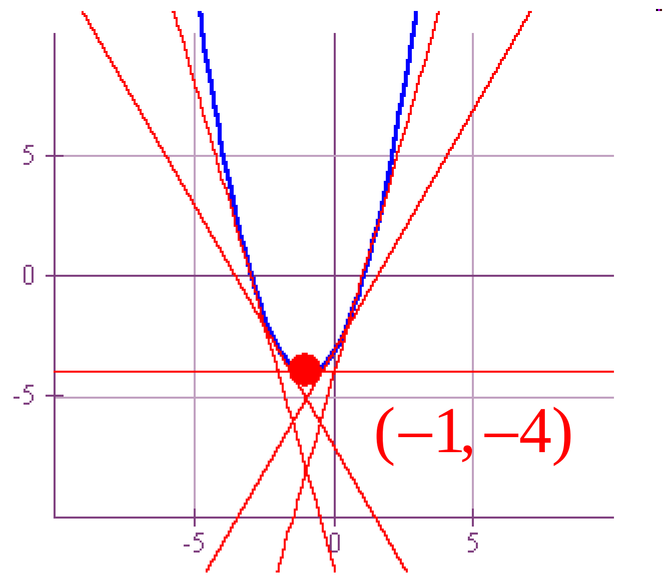
$$f(x) = x^2 + 2x + 3$$

$$f'(x) = 2x + 2$$

$$f'(x) < 0 \text{ if } x < -1$$

$$f'(x) = 0 \text{ if } x = -1$$

$$f'(x) > 0 \text{ if } x > -1$$



THE FIRST DERIVATIVE TEST: Let (a,b) be a critical point for a function $y = f(x)$.

Then,

1. The point (a,b) is a relative minimum point if $f'(x) < 0$ for $x < a$ and $f'(x) > 0$ for $x > a$.

2. The point (a,b) is a relative maximum point if $f'(x) > 0$ for $x < a$ and $f'(x) < 0$ for $x > a$.

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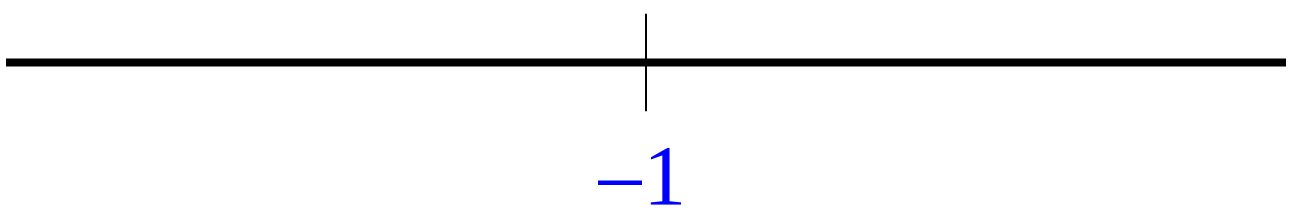

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$f'(x) < 0$ $\quad -1 \quad$ $f'(x) > 0$

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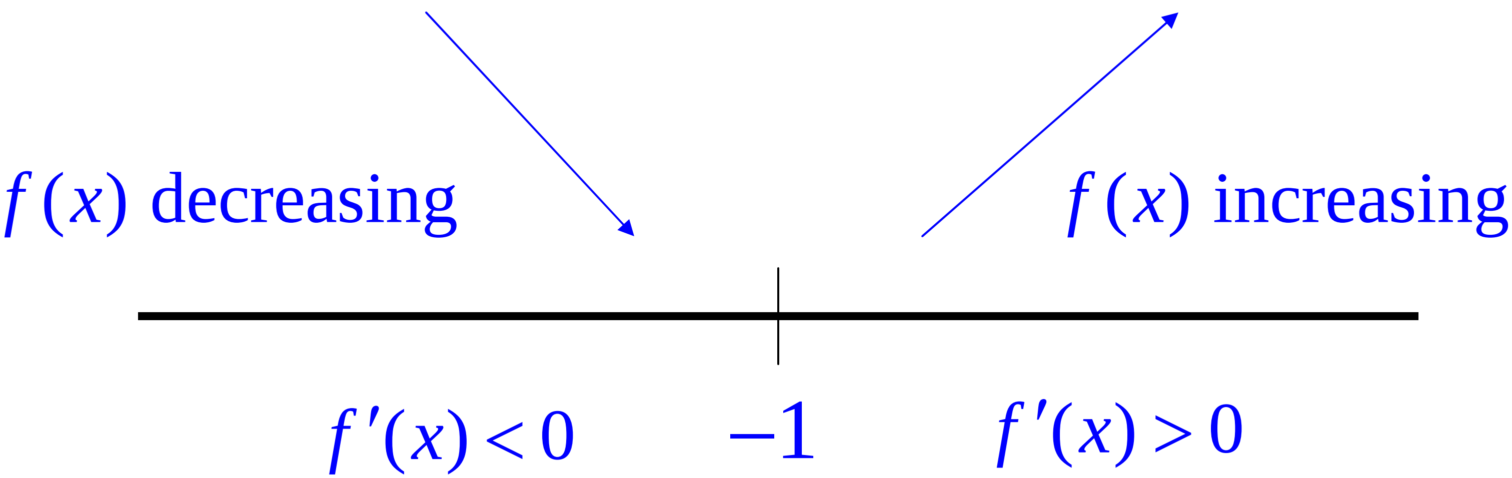

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$$f(-1) = 1 - 2 - 3 = -4$$

relative minimum

$f(x)$ decreasing

$f(x)$ increasing

$f'(x) < 0$ $\quad -1 \quad$ $f'(x) > 0$