# LOGISTIC FUNCTIONS – ANSWERS

(1-4) For each logistic function below, identify $\lim\limits_{x \to \infty} f(x)$, $\lim\limits_{x \to -\infty} f(x)$, **graph** $f(x)$, **and graph the horizontal asymptote corresponding to the upper limit value.**

1. $f(x) = \dfrac{2.5}{1 + 3^{-x}}$

$\lim\limits_{x \to \infty} f(x) = 2.5$

$\lim\limits_{x \to -\infty} f(x) = 0$
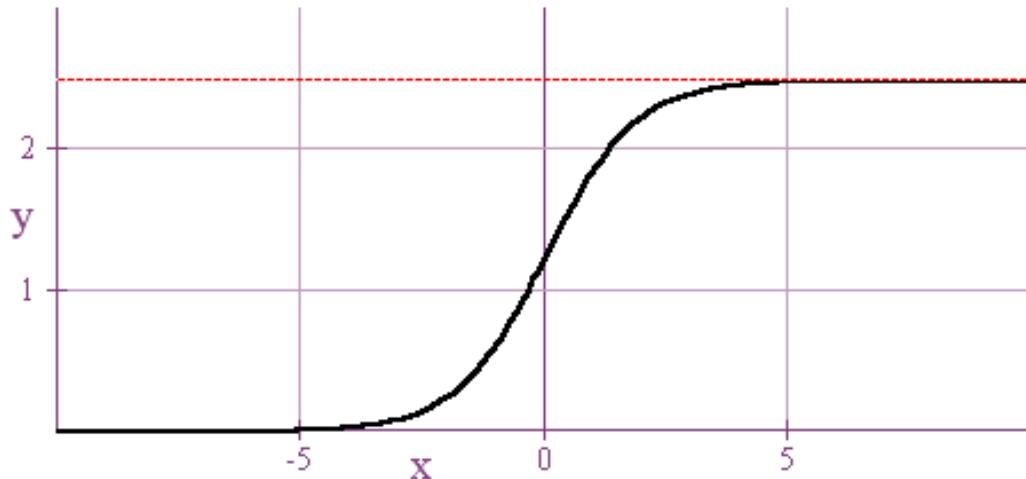
2. $f(x) = \dfrac{1.5}{1+\left(\dfrac{1}{3}\right)^{-x}} = \dfrac{1.5}{1+3^x}$

$\lim\limits_{x \to \infty} f(x) = 0$

$\lim\limits_{x \to -\infty} f(x) = 1.5$
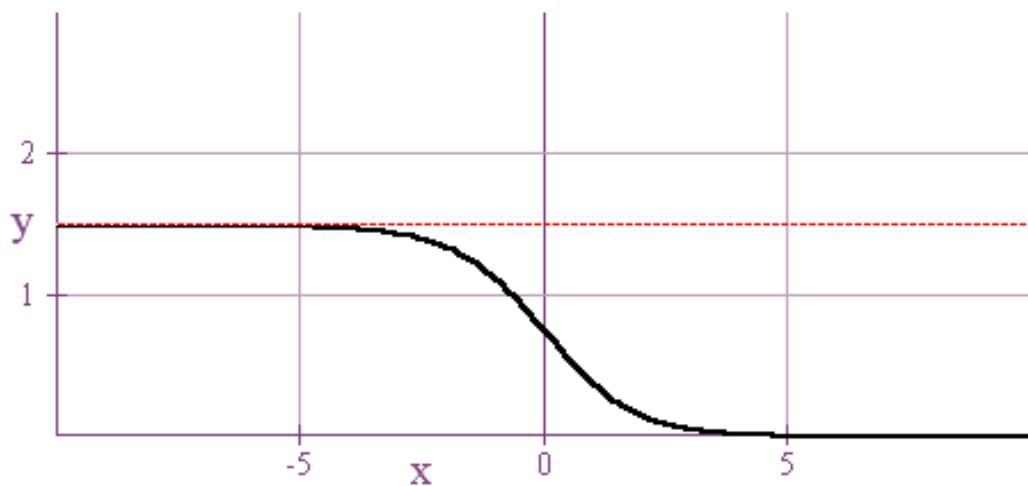
3. $f(x) = \dfrac{3.5}{1+e^{-x}}$

$\lim\limits_{x \to \infty} f(x) = 3.5$

$\lim\limits_{x \to -\infty} f(x) = 0$
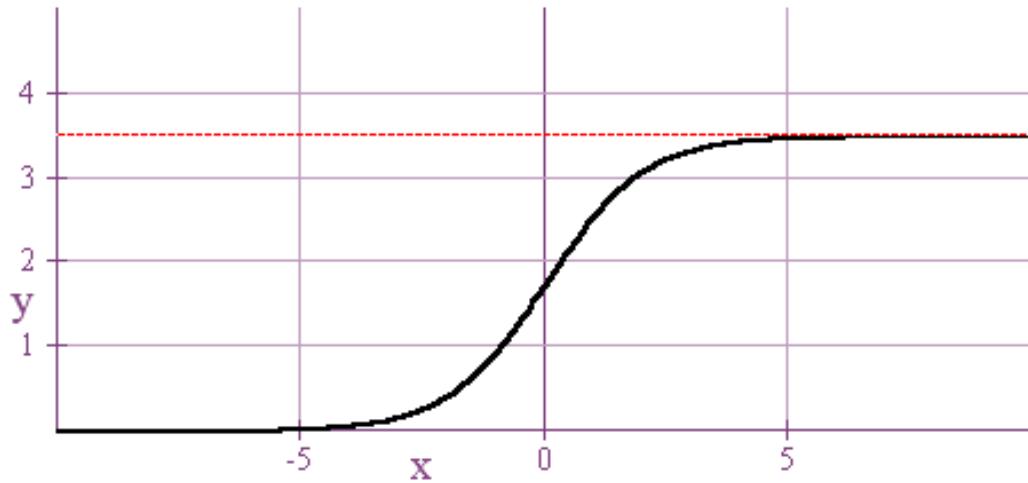
4. $f(x) = \dfrac{4.5}{1+2e^x}$

$\lim\limits_{x\to\infty} f(x) = 0$

$\lim\limits_{x\to-\infty} f(x) = 4.5$
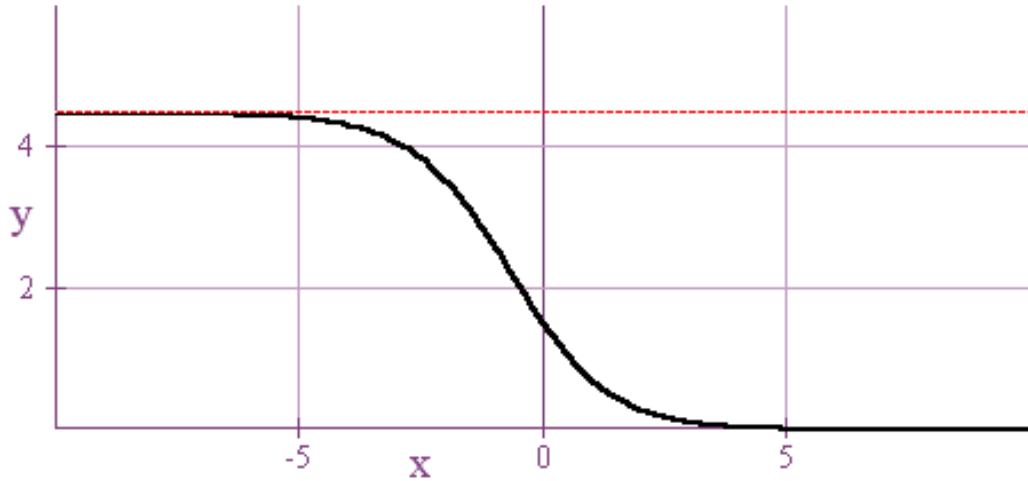
5. Find the logistic function in the form $f(x) = \dfrac{C}{1+A\cdot B^{-x}}$ if it has an upper limit value of 6 and passes through the points $(0,3)$ and $(1,4)$.

The statement of the problem tells us that $C = 6$. If we evaluate the function at 0, then we get $f(0) = \dfrac{6}{1+A} = 3 \Rightarrow 6 = 3 + 3A \Rightarrow 3A = 3 \Rightarrow A = 1$. If we now evaluate our function at 1, we obtain

$f(1) = \dfrac{6}{1+B^{-1}} = 4 \Rightarrow 6 = 4 + 4B^1 \Rightarrow 2 = 4B^1 \Rightarrow B^{-1} = \dfrac{1}{B} = \dfrac{2}{4} = \dfrac{1}{2} \Rightarrow B = 2$.

Therefore, $f(x) = \dfrac{6}{1+2^{-x}}$.

6. Find the logistic function in the form $f(x) = \dfrac{c}{1 + ae^{-bx}}$ that best fits the data below.

Round a, b, and c to the nearest hundredth.

x	0	20	40	60	80	100
y	2.1	3.6	5.0	6.1	6.8	6.9

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L1       L2      L3      2
0        2.1     ------
20       3.6
40       5
60       6.1
80       6.8
100      6.9
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L2(1)=2.1
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Logistic
y=c/(1+ae^(-bx)
a=2.421084529
b=.0435693002
c=7.18999638
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$$f(x) = \frac{7.19}{1 + 2.42e^{-0.04x}}$$