INTEGRATION BY PARTS EXERCISES – ANSWERS

1. $\int \ln x \, dx$

$u = \ln x$
$dv = dx$
$v = x$
$du = \frac{1}{x} dx$

$$uv - \int v du = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - \int 1 dx = x \ln x - x + c$$

2. $\int 5xe^x \, dx$

$u = 5x$
$dv = e^x dx$
$v = e^x$
$du = 5dx$

$$uv - \int v du = 5xe^x - \int 5e^x dx = 5xe^x - 5e^x + c$$

3. $\int (x+1)e^x \, dx$

$u = x + 1$
$dv = e^x dx$
$v = e^x$
$du = dx$

$$uv - \int v du = (x+1)e^x - \int e^x dx = xe^x + e^x - e^x + c = xe^x + c$$

4. $\int xe^{-x} \, dx$

$u = x$
$dv = e^{-x} dx$
$v = -e^x$
$du = dx$

$$uv - \int v du = -xe^{-x} - \int -e^{-x} dx = -xe^{-x} - e^{-x} + c$$

5. $\int x \ln x \, dx$

$u = \ln x$

$dv = xdx$

$v = \frac{x^2}{2}$ $\qquad uv - \int v du = \frac{x^2 \ln x}{2} - \int \frac{x}{2} dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + c$

$du = \frac{1}{x} dx$

6. If a product has a marginal cost given by $\frac{dC}{dx} = (x+1)^2 \ln(x+1)$ and a fixed cost given by $C(0) = \$99$, then find the cost function $C(x)$. Round, as needed, to the nearest cent.

$\int (x+1)^2 \ln(x+1) \, dx$

$u = \ln(x+1)$

$dv = (x+1)^2 dx$

$v = \frac{(x+1)^3}{3}$

$du = \frac{1}{x+1} dx$

$C(x) = uv - \int v du = \frac{(x+1)^3 \ln(x+1)}{3} - \int \frac{(x+1)^2}{3} dx = \frac{(x+1)^3 \ln(x+1)}{3} - \frac{(x+1)^3}{9} + c$

$C(0) = 99 = -\frac{1}{9} + c \Rightarrow c = 99.11$

$\Rightarrow C(x) = \frac{(x+1)^3 \ln(x+1)}{3} - \frac{(x+1)^3}{9} + 99.11$ dollars