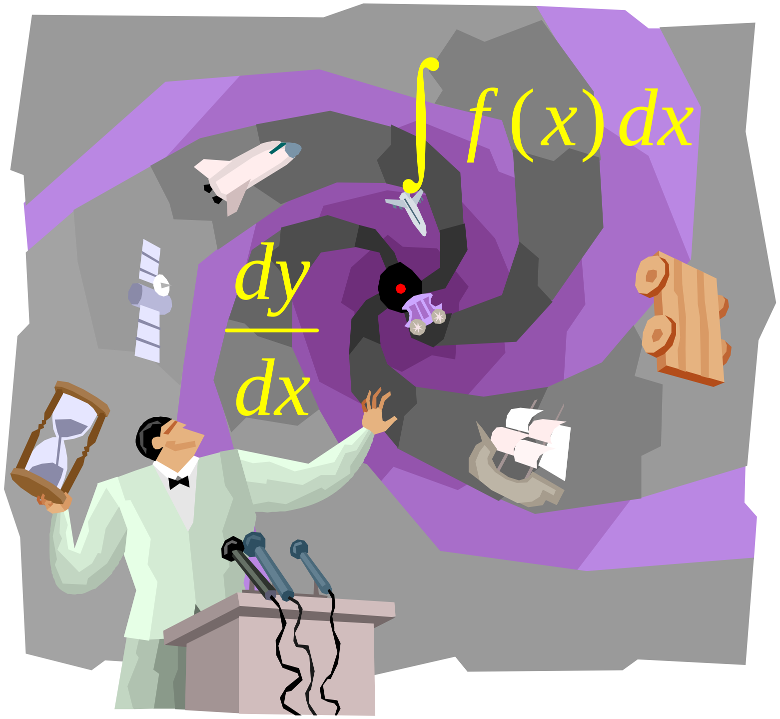
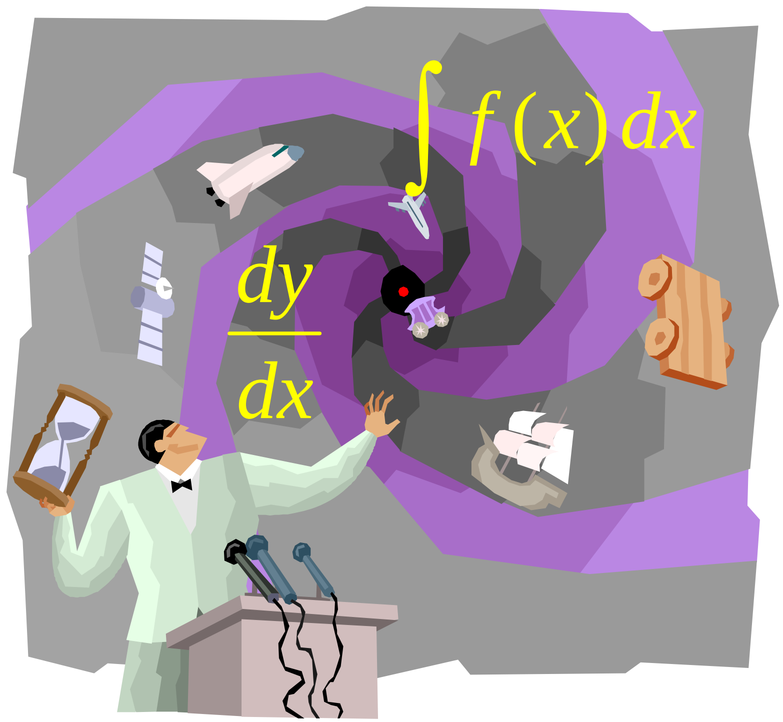
INTEGRATION BY PARTS

When I was a freshman in college, I took differential and integral calculus.



I loved doing the homework problems!



However, one night there was one problem I just couldn't work.

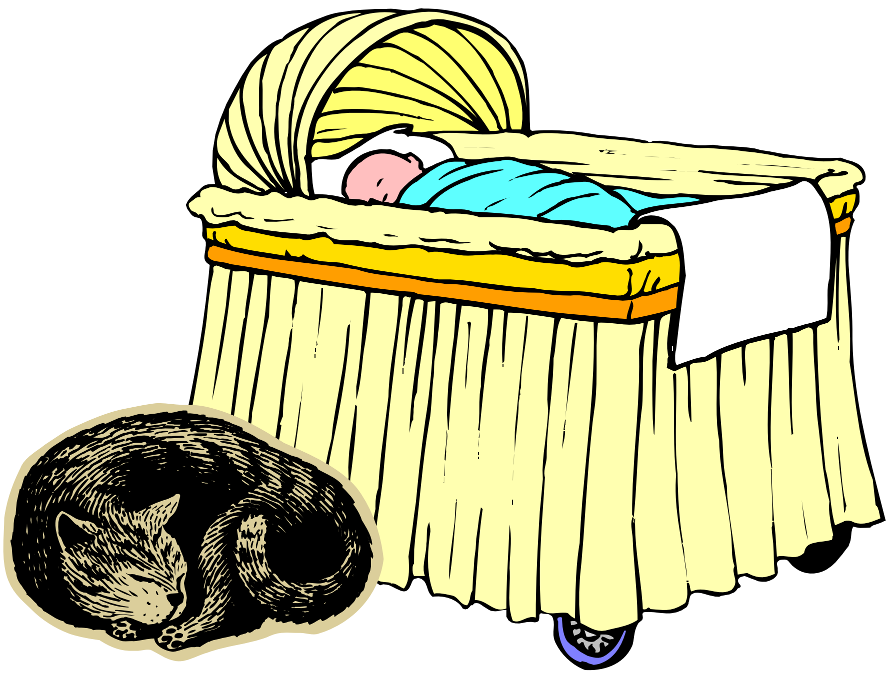
$$\int xe^x \, dx$$

I tried everything, but nothing seemed to work.

$$\int xe^{x}\,dx$$

Finally, I went to bed.

$$\int xe^{x}\,dx$$



But as I was falling asleep, I had a vision!

$$\int xe^{x}\,dx$$

I saw the product formula floating before me.

$$(uv)' = uv' + vu'$$

Then I saw the terms rearrange themselves.

$$(uv)' = uv' + vu'$$

$$uv' = (uv)' - vu'$$

Then I saw integral signs appear.

$$(uv)' = uv' + vu'$$

$$uv' = (uv)' - vu'$$

$$\int u \frac{dv}{dx} dx = \int \frac{d(uv)}{dx} dx - \int v \frac{du}{dx} dx$$

This last expression can be simplified as follows.

$$\int u \frac{dv}{dx} dx = \int \frac{d(uv)}{dx} dx - \int v \frac{du}{dx} dx$$

$$\int u\, dv = uv - \int v\, du$$

So now I knew what I had to do. Identify *u* and *dv*, and then use the formula on the right to try and simplify my original problem.

$$\int u\,dv = uv - \int v\,du$$

$$\int xe^x\,dx$$

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$$\int u\,dv = uv - \int v\,du$$

$$\int xe^x\,dx$$

$$u = x$$

$$dv = e^x\,dx$$

$$v = e^x$$

$$du = dx$$

So now I knew what I had to do. Identify *u* and *dv*, and then use the formula on the right to try and simplify my original problem.

$$\int u\,dv = uv - \int v\,du$$

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + c$$

$$u = x$$

$$dv = e^x\,dx$$

$$v = e^x$$

$$du = dx$$

I was elated! I felt that I had discovered a new and powerful tool for integration. I would be famous!

$$\int u\,dv = uv - \int v\,du$$

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + c$$

$$u = x$$

$$dv = e^x\,dx$$

$$v = e^x$$

$$du = dx$$

However, the next day my professor wrote the formula I had discovered on the board and said, “This is what we call integration by parts.”

$$\int u\,dv = uv - \int v\,du$$

And I’m still not famous.

$$\int u\,dv = uv - \int v\,du$$

And I’m still not famous.

… At least not for that formula!

$$\int u\,dv = uv - \int v\,du$$

Here's another example.

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln x\,dx$$

You want to set *dv* equal to something you can find an antiderivative of, and then set *u* equal to whatever is left.

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You want to set *dv* equal to something you can find an antiderivative of, and then set *u* equal to whatever is left.

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln x\,dx$$

$$u = \ln x$$

$$dv = dx$$

Next, integrate to find *v* and differentiate to find *du*.

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln x\,dx$$

$$u = \ln x$$

$$dv = dx$$

$$v = x$$

$$du = \frac{1}{x}dx$$

And now use the formula.

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln x\,dx = uv - \int v\,du = x\ln x - \int x\cdot\frac{1}{x}dx = x\ln x - \int dx$$

$$= x\ln x - x + c$$

$$u = \ln x$$

$$dv = dx$$

$$v = x$$

$$du = \frac{1}{x}dx$$

And now use the formula.

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln x\,dx = uv - \int v\,du = x\ln x - \int x\cdot\frac{1}{x}dx = x\ln x - \int dx$$

$$= x\ln x - x + c$$

$u = \ln x$

$dv = dx$

$v = x$

$du = \frac{1}{x}dx$

Super Cool!