IMPROPER INTEGRAL EXERCISES – ANSWERS

1. Evaluate $\int_1^\infty \frac{1}{x}\,dx$.

$$\int_1^\infty \frac{1}{x}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x}\,dx = \lim_{t\to\infty} \ln|x|\Big|_1^t = \lim_{t\to\infty}\left(\ln t - \ln 1\right) = \lim_{t\to\infty}\ln t = \infty$$

2. Evaluate $\int_1^\infty \frac{1}{x^2}\,dx$.

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^2}\,dx = \lim_{t\to\infty}\frac{-1}{x}\Big|_1^t = \lim_{t\to\infty}\left(\frac{-1}{t} - \frac{-1}{1}\right) = 1$$

3. Evaluate $\int_1^\infty \frac{1}{x^3}\,dx$.

$$\int_1^\infty \frac{1}{x^3}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^3}\,dx = \lim_{t\to\infty}\frac{-1}{2x^2}\Big|_1^t = \lim_{t\to\infty}\left(\frac{-1}{2t^2} - \frac{-1}{2(1^2)}\right) = \frac{1}{2}$$

4. Evaluate $\int_1^\infty \frac{1}{x^4}\,dx$.

$$\int_1^\infty \frac{1}{x^4}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^4}\,dx = \lim_{t\to\infty}\frac{-1}{3x^3}\Big|_1^t = \lim_{t\to\infty}\left(\frac{-1}{3t^3} - \frac{-1}{3(1^3)}\right) = \frac{1}{3}$$

5. Evaluate $\int_1^\infty \frac{1}{x^5}\,dx$.

$$\int_1^\infty \frac{1}{x^5}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^5}\,dx = \lim_{t\to\infty}\frac{-1}{4x^4}\Big|_1^t = \lim_{t\to\infty}\left(\frac{-1}{4t^4} - \frac{-1}{4(1^3)}\right) = \frac{1}{4}$$

6. Suppose you go to a pharmacy to pick up a prescription and the average waiting time is 5 minutes. It can be shown that the probability that you will have to wait between a and b minutes is given by the integral $\int_a^b \frac{1}{5} e^{-x/5}\, dx$. What is the probability that you will have to wait from 0 to 10 minutes? Round your answer to two decimal places.

$$\int_0^{10} \frac{1}{5} e^{-x/5}\, dx = -e^{-x/5}\Big|_0^{10} = -e^{-10/5} + e^0 = 1 - e^{-2} \approx 0.86$$

7. Suppose you go to a pharmacy to pick up a prescription and the average waiting time is 5 minutes. It can be shown that the probability that you will have to wait between a and b minutes is given by the integral $\int_a^b \frac{1}{5} e^{-x/5}\, dx$. What is the probability that you will have to wait more than 10 minutes? Round your answer to two decimal places.

$$\int_{10}^{\infty} \frac{1}{5} e^{-x/5}\, dx = \lim_{t\to\infty}\left(-e^{-x/5}\right)\Big|_{10}^{t} = \lim_{t\to\infty}\left(-e^{-t/5} + e^{-2}\right) = \lim_{t\to\infty}\left(-\frac{1}{e^{t/5}} + \frac{1}{e^2}\right) \approx 0.14$$