Finding Derivatives Algebraically

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

We'll now focus on finding derivatives by evaluating a limit algebraically. If we are trying to find the derivative at a single point *a*, then the first formula below is the easiest one to use.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

EXAMPLE: Let ***f(x)=x²*** **and let** ***a=1.***

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

EXAMPLE: Let $f(x)=x^2$ and let $a=1$.

$$f'(1)=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1}\frac{x^2-1}{x-1}=\lim_{x\to 1}\frac{(x+1)(x-1)}{(x-1)}$$

$$=\lim_{x\to 1}(x+1)=1+1=2$$

EXAMPLE: Let *$f(x)=x^2$* and let *$a=1$*.

Thus, the slope of the tangent line is 2 when *$a=1$*, and this is also the *instantaneous rate of change* at this point.

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$$=\lim_{x\to 1}(x+1)=1+1=2$$

EXAMPLE: Let *$f(x)=x^3-x$* and let *$a=1$.*

$$f'(1)=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1}\frac{(x^3-x)-0}{x-1}=\lim_{x\to 1}\frac{x(x^2-1)}{(x-1)}$$

$$=\lim_{x\to 1}\frac{x(x+1)(x-1)}{(x-1)}=\lim_{x\to 1}x(x+1)=1\cdot(1+1)=2$$

EXAMPLE: Let *$f(x)=x^3-x$* and let *$a=1$.*

Here we had a different function, but by coincidence the derivative at 1 still came out equal to 2.

$$f'(1)=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1}\frac{(x^3-x)-0}{x-1}=\lim_{x\to 1}\frac{x(x^2-1)}{(x-1)}$$

$$=\lim_{x\to 1}\frac{x(x+1)(x-1)}{(x-1)}=\lim_{x\to 1}x(x+1)=1\cdot(1+1)=2$$

If instead of evaluating the derivative at a specific point, we want to find a general formula for the derivative, then the second formula is the best to use.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

EXAMPLE: Let ***$f(x)=x^2$*** **and find** ***$f'(x)$.***

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

EXAMPLE: Let *f(x)=x²* and find *f'(x)*.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2hx + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0}(2x+h) = 2x + 0 = 2x$$

EXAMPLE: Let *$f(x)=x^2$* and find *$f'(x)$*.

The advantage now is that we can use this formula to quickly find the *derivative/instantaneous rate of change/ slope of tangent line* at any point.

$$f'(x) = 2x$$

$$f'(1) = 2$$

$$f'(2) = 4$$

$$f'(3) = 6$$

EXAMPLE: Suppose *$f(x)=x^2$* represents the number of miles you have walked after *x hours*. Find your *instantaneous velocity* after *0, .5, 1.5,* and *2 hours.*

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$$f'(x) = 2x$$

$$f'(0) = 0 \text{ miles/hour}$$

$$f'(.5) = 1 \text{ miles/hour}$$

$$f'(1.5) = 3 \text{ miles/hour}$$

$$f'(2) = 4 \text{ miles/hour}$$

EXAMPLE: Let ***f(x)=x³*** **and find** ***f'(x).***

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

EXAMPLE: Let *$f(x)=x^3$* and find *$f'(x)$*.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$$

$$=\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}$$

$$=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}=\lim_{h\to 0}(3x^2+3xh+h^2)$$

$$=3x^2+3x\cdot 0+0^2=3x^2$$

EXAMPLE: Let *$f(x)=x^3$* and find *$f'(x)$*.

Again, we now have a formula that we can use to quickly find the derivative at several points.

$$f'(x) = 3x^2$$

$$f'(1) = 3 \cdot 1^2 = 3$$

$$f'(2) = 3 \cdot 2^2 = 12$$

$$f'(3) = 3 \cdot 3^2 = 27$$

EXAMPLE: Let *$f(x)=x^3$* and find *$f'(x)$*.

Again, we now have a formula that we can use to quickly find the derivative at several points.

$$f'(x) = 3x^2$$

Amazing!

$$f'(1) = 3 \cdot 1^2 = 3$$

$$f'(2) = 3 \cdot 2^2 = 12$$

$$f'(3) = 3 \cdot 3^2 = 27$$