DERIVATIVE FORMULAS 3

THEOREM: If $y = e^x$, then $\frac{dy}{dx} = e^x$.

ARGUMENT: Below is the graph of $y = e^x$, a typical exponential growth function, and next to it is another graph of $y = e^x$ with several tangent lines attached.

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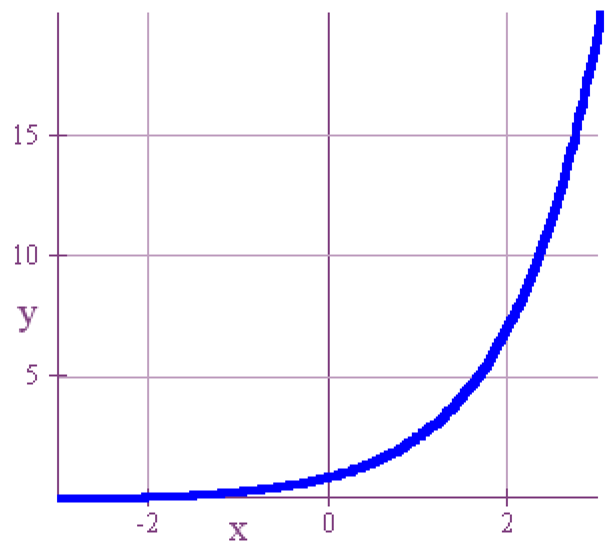

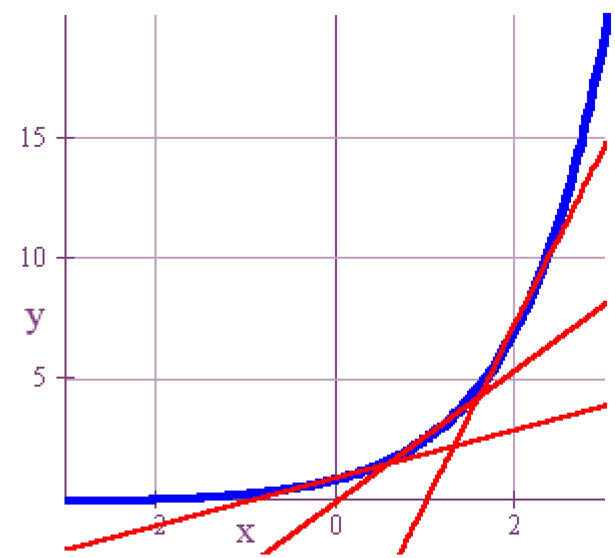

Notice that just as our original function is increasing exponentially, so also do the slopes of the tangent lines seem to be increasing exponentially. This suggests that the derivative of an exponential function is another exponential function!

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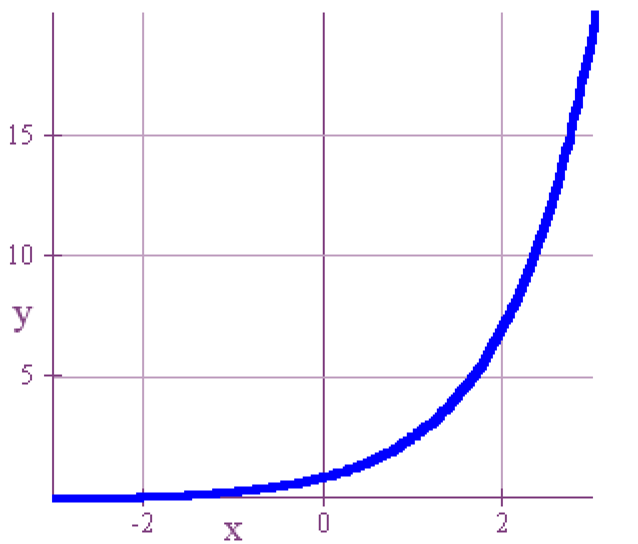

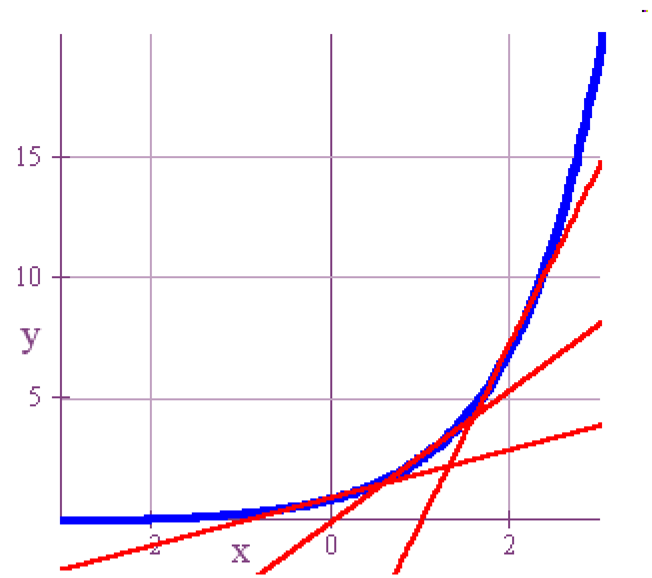

Below now is the graph of $y = 1.5^x$ (in blue)along with the graph of its derivative in red. Notice that the graph of the derivative function is below the graph of our original function.

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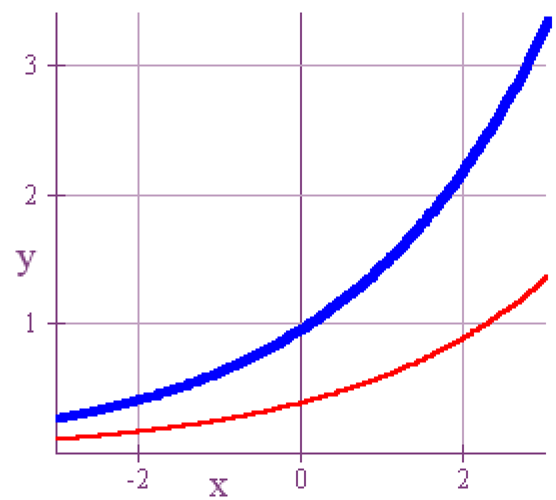

Next we have the graph of $y = 4.5^x$ (in blue) along with the graph of its derivative in red. Notice that the graph of the derivative function is now above our original function.

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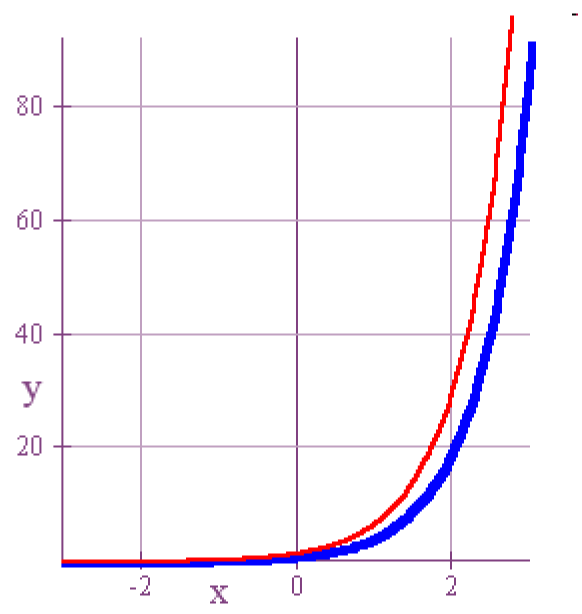

These results suggest that somewhere between 1.5 and 4.5 there is a base for which the graph of the exponential function and its derivative function are one and the same. Guess what base that happens at?

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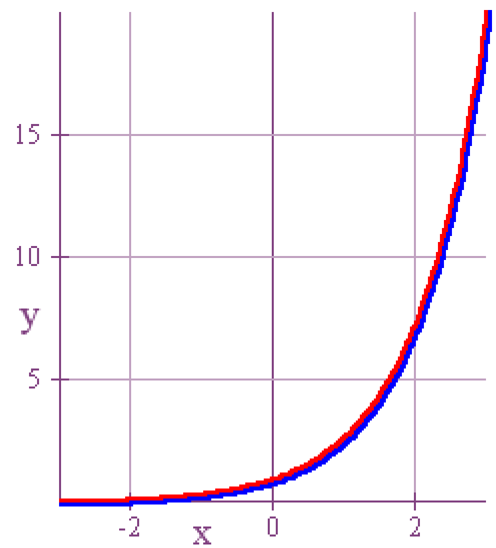

That happens if the base is *e*.

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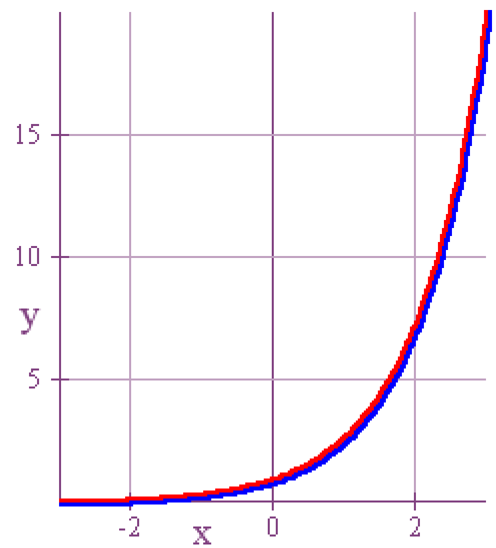

Thus, the derivative of e^x is e^x.

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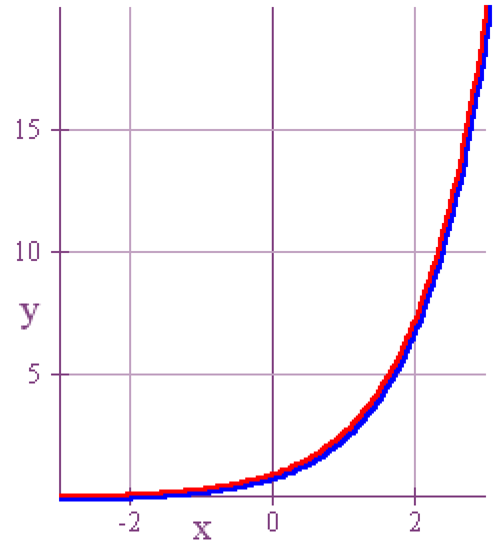
$$y = e^x$$

$$\frac{dy}{dx} = e^x$$



For derivatives of exponential functions with a base other than *e*, we have the following theorem.

THEOREM: $\dfrac{d(b^x)}{dx} = b^x \cdot \ln b.$

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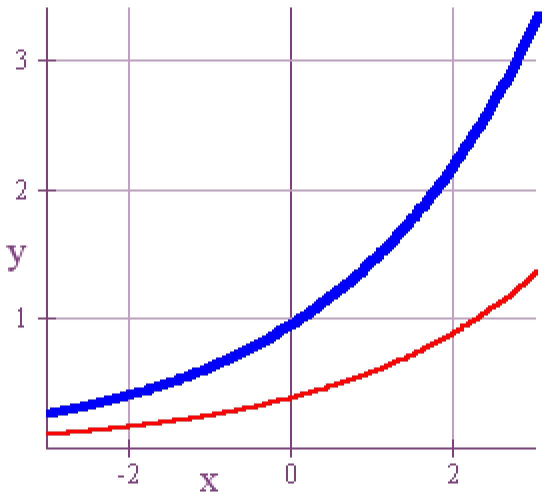
$$y = 1.5^x$$

$$\frac{dy}{dx} = 1.5^x \cdot \ln 1.5$$



For derivatives of exponential functions with a base other than *e*, we have the following theorem.

THEOREM: $\dfrac{d(b^x)}{dx} = b^x \cdot \ln b.$

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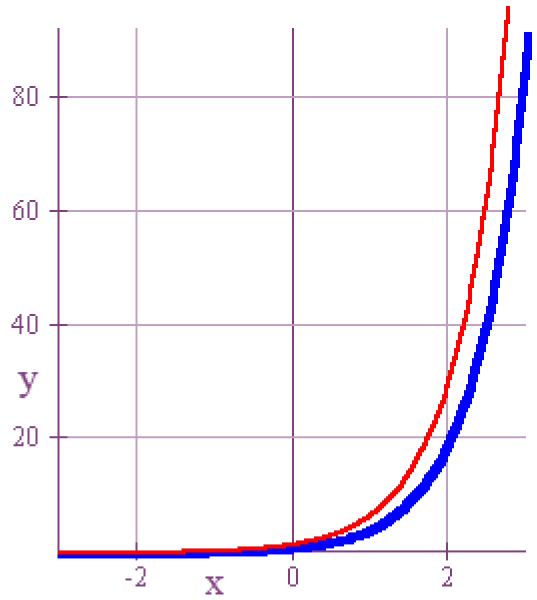
$$y = 4.5^x$$

$$\frac{dy}{dx} = 4.5^x \cdot \ln 4.5$$



THEOREM: If $y = \ln x, x > 0$, then $\dfrac{dy}{dx} = \dfrac{1}{x}$.

PROOF: Suppose $y = \ln x$. Then $e^y = e^{\ln x} = x$. Hence,

$$\frac{de^y}{dx} = e^y \cdot \frac{dy}{dx} = e^{\ln x} \cdot \frac{d(\ln x)}{dx} = x \cdot \frac{d(\ln x)}{dx} = 1 \Rightarrow \frac{d(\ln x)}{dx} = \frac{1}{x}.$$

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THEOREM: If $y = \ln|x|, x \neq 0,$ then $\dfrac{dy}{dx} = \dfrac{1}{x}.$

PROOF: Suppose $y = \ln|x|$ and $x > 0$. Then

$\dfrac{d \ln|x|}{dx} = \dfrac{d \ln x}{dx} = \dfrac{1}{x}.$ Now suppose $x < 0$. Then

$$\frac{d \ln|x|}{dx} = \frac{d \ln(-x)}{dx} = \frac{1}{-x} \cdot (-1) = \frac{1}{x}.$$

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SUMMARY 3

1. $$\frac{d(c)}{dx} = 0$$

2. $$\frac{dx^n}{dx} = nx^{n-1}$$

3. $$\frac{d(cf(x))}{dx} = c\frac{df}{dx}$$

4. $$\frac{d(f \pm g)}{dx} = \frac{df}{dx} \pm \frac{dg}{dx}$$

5. $$\frac{df(g(x))}{dx} = \frac{df}{dg}\frac{dg}{dx}$$

6. $$\frac{d(fg)}{dx} = f\frac{dg}{dx} + g\frac{df}{dx}$$

7. $$\frac{d\left(\frac{f}{g}\right)}{dx} = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$$

8. $$\frac{de^x}{dx} = e^x, \quad \frac{db^x}{dx} = b^x \ln b$$

9. $$\frac{d\ln x}{dx} = \frac{1}{x}, \quad \frac{d\log_b x}{dx} = \frac{1}{x\ln b}, \; x > 0$$

10. $$\frac{d\ln|x|}{dx} = \frac{1}{x}, \; x \neq 0$$

EXAMPLES:

$$y = e^{x}$$

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$$y = e^x$$

$$\frac{dy}{dx} = e^x$$

EXAMPLES:

$$y = 2^x$$

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$$y = 2^x$$

$$\frac{dy}{dx} = 2^x \cdot \ln 2$$

EXAMPLES:

$$y = e^{x^2 + 4x}$$

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$$y = e^{x^2+4x}$$

$$\frac{dy}{dx} = e^{x^2+4x}(2x+4)$$

EXAMPLES:

$$y = \ln(x^2 + 4x)$$

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$$\frac{dy}{dx} = \frac{1}{x^2 + 4x} \cdot (2x + 4) = \frac{2x + 4}{x^2 + 4x}$$

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$$y = \ln\left|x^2 + 4x\right|$$

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$$\frac{dy}{dx} = \frac{1}{(x^2 + 4x)\ln 10} \cdot (2x + 4) = \frac{2x + 4}{(x^2 + 4x)\ln 10}$$