DERIVATIVE FORMULAS 1

THEOREM: If $f(x) = c$ is a constant function, then $f'(x) = 0$.

PROOF: Suppose $f(x) = c$ is a constant function. Then

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = 0.$$

THEOREM: If $f(x) = x$, then $f'(x) = 1$.

PROOF: Suppose $f(x) = x$. Then

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{x+h-x}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1.$$

THEOREM: If $f(x) = x^2$, then $f'(x) = 2x$.

PROOF: Suppose $f(x) = x^2$. Then

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{2hx + h^2}{h} = \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0} (2x+h) = 2x.$$

THEOREM: If $f(x) = x^3$, then $f'(x) = 3x^2$.

PROOF: Suppose $f(x) = x^3$. Then

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 - x^3}{h} = \lim_{h \to 0} \frac{3hx^2 + 3h^2x + h^3}{h}$$

$$= \lim_{h \to 0} \frac{h(3x^2 + 3hx + h^2)}{h} = \lim_{h \to 0} (3x^2 + 3hx + h^2) = 3x^2.$$

THEOREM: If $f(x) = x^n$ where n is any real number, then $f'(x) = nx^{n-1}$.

ARGUMENT: This is not a formal proof. We are simply going to recognize the pattern. In particular, if we write $1 = x^0$, then we've seen the following results:

$$\frac{d(x^0)}{dx} = \frac{d(1)}{dx} = 0 = 0 \cdot x^{-1}$$

$$\frac{d(x^1)}{dx} = 1 = 1 \cdot x^0$$

$$\frac{dx^2}{dx} = 2x = 2x^1$$

$$\frac{dx^3}{dx} = 3x^2$$

At this point, we'll take it on faith that the pattern holds for all real numbers n.

THEOREM: If $f(x) = c \cdot g(x)$ where c is a constant, then $f'(x) = c \cdot g'(x)$.

PROOF: Suppose $f(x) = c \cdot g(x)$ where c is a constant. Then

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c \cdot g(x+h) - c \cdot g(x)}{h}$$

$$= c \cdot \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = c \cdot g'(x).$$

THEOREM: If $y = (f+g)(x) = f(x) + g(x)$,

then $\dfrac{dy}{dx} = f'(x) + g'(x)$.

PROOF: Suppose $y = (f+g)(x) = f(x) + g(x)$. Then

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{(f+g)(x+h) - (f+g)(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} = f'(x) + g'(x).$$

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PROOF: Suppose $y = (f+g)(x) = f(x) + g(x)$. Then

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{(f+g)(x+h) - (f+g)(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} = f'(x) + g'(x).$$

COROLLARY: If $y = (f-g)(x) = f(x) - g(x)$,

then $\dfrac{dy}{dx} = f'(x) - g'(x)$.

SUMMARY 1

1. $\dfrac{d(c)}{dx} = 0$

2. $\dfrac{dx^n}{dx} = nx^{n-1}$

3. $\dfrac{d(cf(x))}{dx} = c\dfrac{df}{dx}$

4. $\dfrac{d(f \pm g)}{dx} = \dfrac{df}{dx} \pm \dfrac{dg}{dx}$

EXAMPLES:

$$y = 2$$

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$$y = 2$$

$$\frac{dy}{dx} = 0$$

EXAMPLES:

$$y = x = x^1$$

EXAMPLES:

$$y = x = x^1$$

$$\frac{dy}{dx} = 1 \cdot x^0 = 1 \cdot 1 = 1$$

EXAMPLES:

$$y = x^{10}$$

EXAMPLES:

$$y = x^{10}$$

$$\frac{dy}{dx} = 10x^9$$

EXAMPLES:

$$y = 5x^3$$

EXAMPLES:

$$y = 5x^3$$

$$\frac{dy}{dx} = 5 \cdot 3x^2 = 15x^2$$

EXAMPLES:

$$y = x^2 - 3x + 1$$

EXAMPLES:

$$y = x^2 - 3x + 1$$

$$\frac{dy}{dx} = 2x - 3$$

EXAMPLES:

$$y = 4x^2 + 8x + 5$$

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$$y = 4x^2 + 8x + 5$$

$$\frac{dy}{dx} = 8x + 8$$

EXAMPLES:

$$y = \frac{1}{x^2} = x^{-2}$$

EXAMPLES:

$$y = \frac{1}{x^2} = x^{-2}$$

$$\frac{dy}{dx} = -2x^{-3} = -\frac{2}{x^3}$$

EXAMPLES:

$$y = \sqrt{x} = x^{1/2}$$

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$$y = \sqrt{x} = x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

EXAMPLES:

$$y = x^{\pi}$$

EXAMPLES:

$$y = x^{\pi}$$

$$\frac{dy}{dx} = \pi x^{\pi - 1}$$