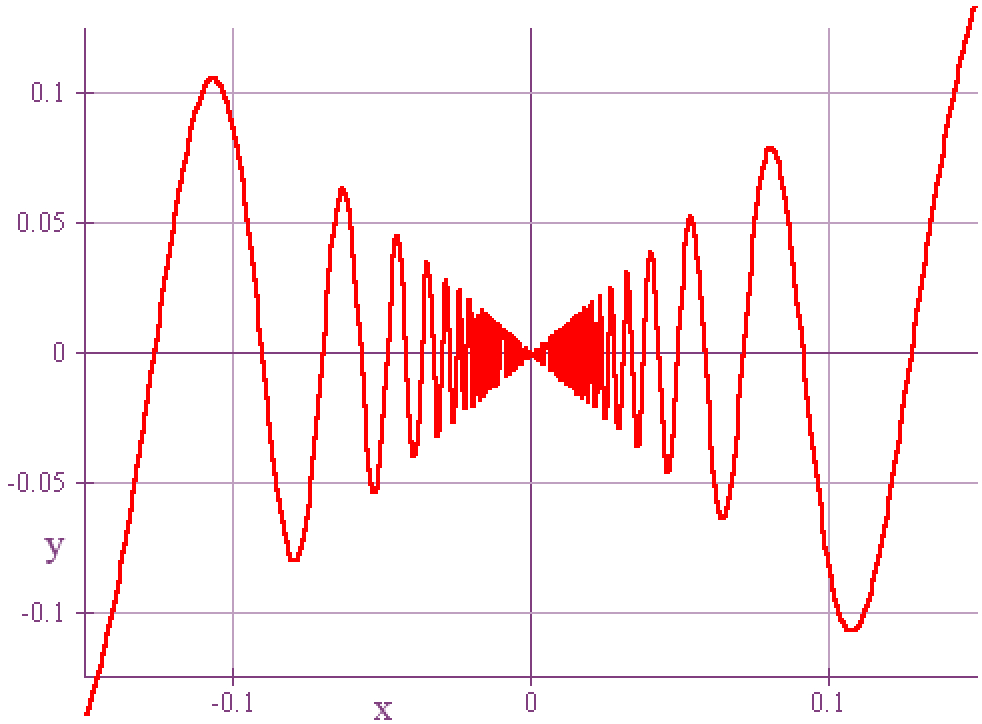
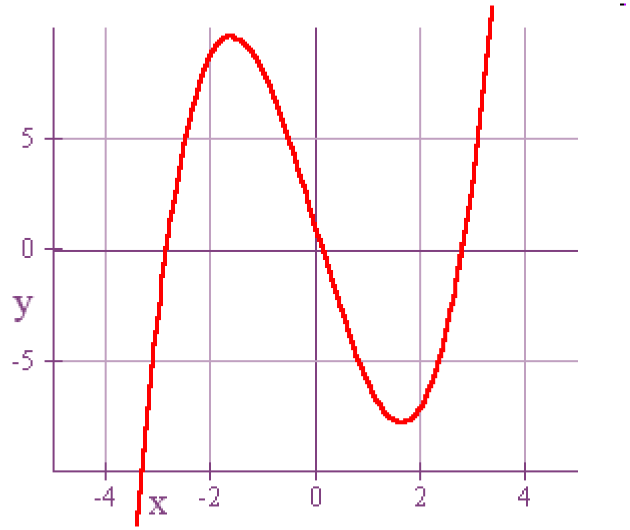
CONTINUITY



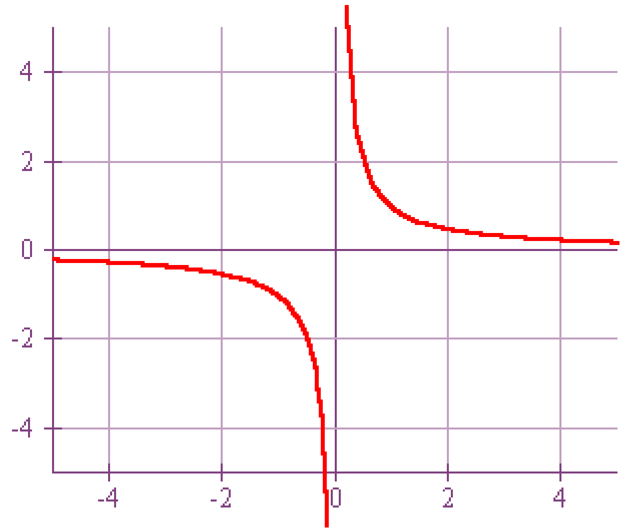
Typically, we think of a graph as being continuous at all real numbers if its graph represents a curve that can be drawn without having to lift one's pencil off the paper.



$$y = x^3 - 8x + 1$$

Continuous at all real numbers

Similarly, a graph is discontinuous at a real number if its graph has a break at that point.



$$y = \frac{1}{x}$$

Discontinuous at *x = 0*

We can define continuity at a real number *a* using the concept of a limit.

DEFINITION: A function $y = f(x)$ is continuous at a real number a if $\lim_{x \to a} f(x) = f(a)$.

We also can make this definition a little easier to grasp by breaking up the condition into three parts.

DEFINITION: A function $y = f(x)$ is continuous at a real number a if:

1. $f(a)$ exists
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

These criteria describe three ways in which a function can fail to be continuous at a real number.

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Let’s now look at a few examples.

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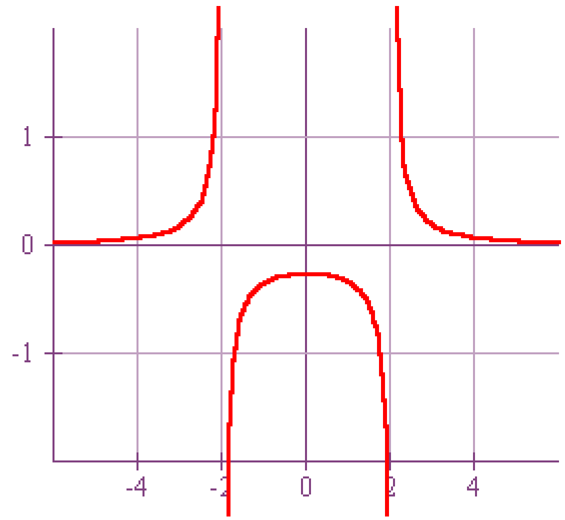
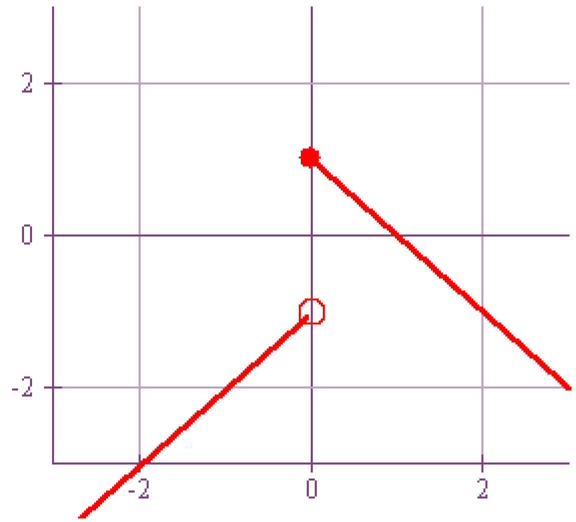
1. $f(x) = \frac{1}{x^2 - 4}$



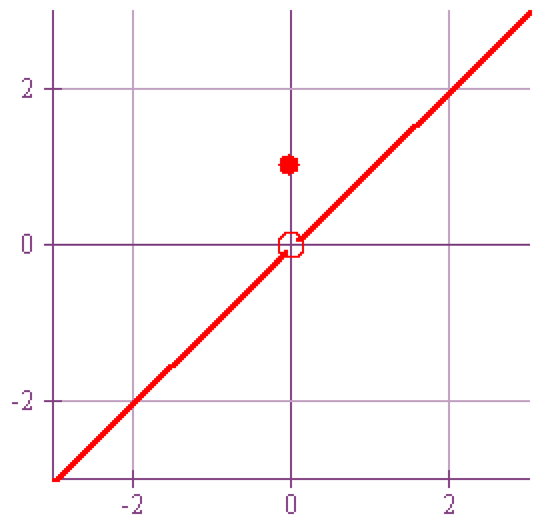
The function is discontinuous at *x = -2* and *x=2* since the function is undefined at these values.

2. $f(x)=\begin{cases} x-1 & \text{if } x<0 \\ -x+1 & \text{if } x\geq 0 \end{cases}$



The function is discontinuous at $x=0$ since $\lim\limits_{x\to 0} f(x)$ does not exist.

3. $f(x) = \begin{cases} x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$



The function is discontinuous at $x = 0$ since $\lim_{x \to 0} f(x) \neq f(0)$.

To find real number values at which a function is not continuous:

1. Examine both the function and its graph.
2. Look for values at which $f(x)$ is not defined.
3. Look for values at which $\lim_{x \to a} f(x)$ is not defined.
4. Look for values at which $\lim_{x \to a} f(x) \neq f(a)$.