

CONCAVITY AND INFLECTION POINT EXERCISES – ANSWERS

For each problem below, identify the intervals over which the function is concave up and the intervals over which the function is concave down. Also give the coordinates of any inflection points. In problem 3, give your answers in decimal form rounded to 3 decimal places. In all other problems, give exact answers in fraction form.

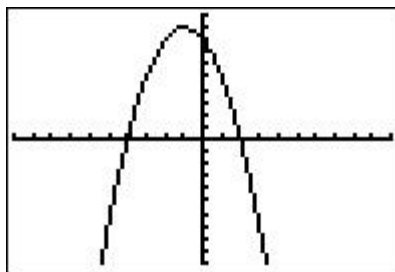
1. $f(x) = -x^2 - 2x + 8$

$$f'(x) = -2x - 2$$

$$f''(x) = -2 < 0$$

\Rightarrow Concave down for all real numbers, $(-\infty, \infty)$.

No inflection points.



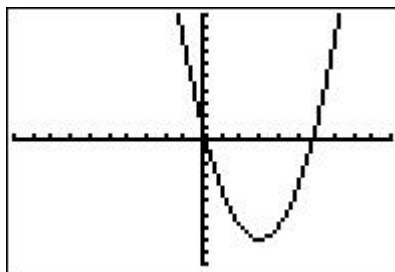
2. $f(x) = x^2 - 6x + 1$

$$f'(x) = 2x - 6$$

$$f''(x) = 2 > 0$$

\Rightarrow Concave up for all real numbers, $(-\infty, \infty)$.

No inflection points.



3. $f(x) = x^4 - 5x^2 + 1$

$$f'(x) = 4x^3 - 10x$$

$$f''(x) = 12x^2 - 10 = 2(6x^2 - 5)$$

$$f''(x) = 0 \Rightarrow 6x^2 - 5 = 0$$

$$\Rightarrow x^2 = \frac{5}{6} \Rightarrow x = \pm \sqrt{\frac{5}{6}} \approx 0.913$$

$$f''(x) > 0 \text{ if } x < -\sqrt{\frac{5}{6}} \text{ or } x > \sqrt{\frac{5}{6}}$$

$$f''(x) < 0 \text{ if } -\sqrt{\frac{5}{6}} < x < \sqrt{\frac{5}{6}}$$

$\Rightarrow f(x)$ is concave up if $x < -0.913$ or

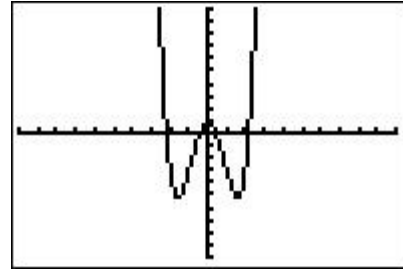
$x > 0.913$. $f(x)$ is concave down if

$-0.913 < x < 0.913$.

Concave up on $(-\infty, -0.913)$ & $(0.913, \infty)$.

Concave down on $(-0.913, 0.913)$.

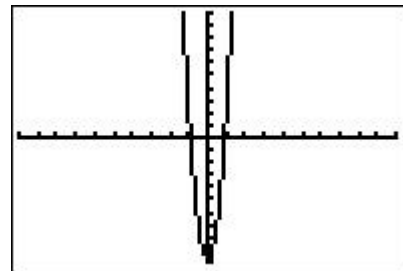
Inflection points are $(-0.913, -2.473)$ & $(0.913, -2.473)$



$$f(x) = x^4 - 5x^2 + 1$$

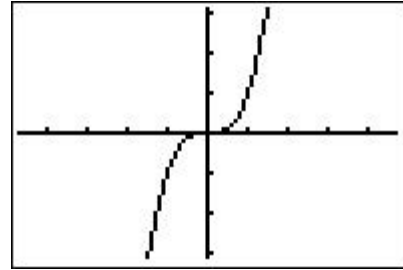


$$f'(x) = 4x^3 - 10x$$



$$f''(x) = 12x^2 - 10$$

4. $f(x) = x^3$
 $f'(x) = 3x^2$
 $f''(x) = 6x$
 $f''(x) = 0 \Rightarrow x = 0$
 $f''(x) < 0$ if $x < 0$ and
 $f''(x) > 0$ if $x > 0$
 Concave down on $(-\infty, 0)$
 Concave up on $(0, \infty)$
 Inflection point at $(0, 0)$



5. $f(x) = x^3 - x^2 - x - 1$
 $f'(x) = 3x^2 - 2x - 1$
 $f''(x) = 6x - 2 = 2(3x - 1)$
 $f''(x) = 0 \Rightarrow x = 1/3$
 $f''(x) < 0$ if $x < 1/3$
 $f''(x) > 0$ if $x > 1/3$
 Concave down on $(-\infty, 1/3)$
 Concave up on $(1/3, \infty)$
 Inflection point at $(1/3, -38/27)$

